Force on upper face of cylinder has magnitude :-

$$F_1 = -P(r+dr) \cdot A$$

Pressure at $r+dr$

$$F_G = -\frac{G M(r)\, dm}{r^2}$$

$M(r)$: mass of atmosphere + planet inside radius $r$

From lecture 2, $\frac{GM(r)}{r^2} \equiv g$ gravitational acceleration

Also, $dm = \rho(r)\, dV = \rho A\, dr$

$dV$: Volume of cylinder

In hydrostatic equilibrium, the NET force on the cylinder is zero. i.e. $F_1 + F_2 = 0$

$$P(r+dr) \cdot A = P(r) \cdot A - \rho g A\, dr$$

$$\Leftrightarrow \quad P(r+dr) - P(r) = -\rho g\, dr$$