Total energy $= K + U = \frac{1}{2} m v_0^2 - \frac{GMm}{R}$

↑ CONSERVED

As the projectile climbs, planet's gravity slows it down; loses K.E. and gains P.E.

- If projectile reaches $v = 0$ at finite r, it falls back to planet.
- If $v \geq 0$ for $r = \infty$ $\Rightarrow$ ESCAPE

LIMITING CASE

$v = 0$ at $r = \infty$ $\Rightarrow$ K.E. $= 0$

BUT $U = 0$ at $r = \infty$ $\Rightarrow$ Total energy $= 0$

Since total energy conserved

$\Rightarrow$ initial total energy $= 0$

$\therefore \frac{1}{2} m v_0^2 = \frac{GMm}{R}$ $\Rightarrow$ $v_0 = \sqrt{\frac{2GM}{R}}$