Orbit determination for extra-solar planetary systems.
Lecture II

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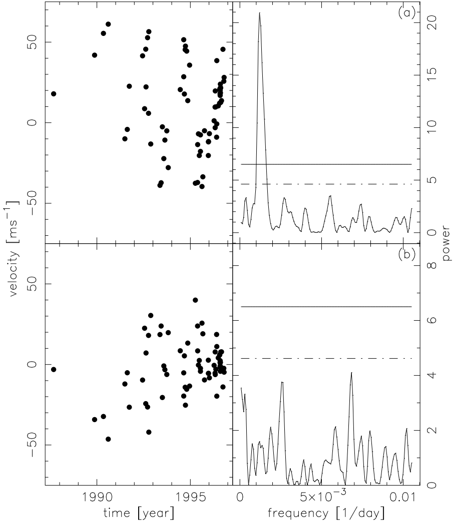
June 3, 2007

Outline

Outline

A planet around 16 Cygni B.

Cochran W. D., Hatzes A. P., Butler R. P., Marcy G. W., 1997, ApJ, 483, 457.



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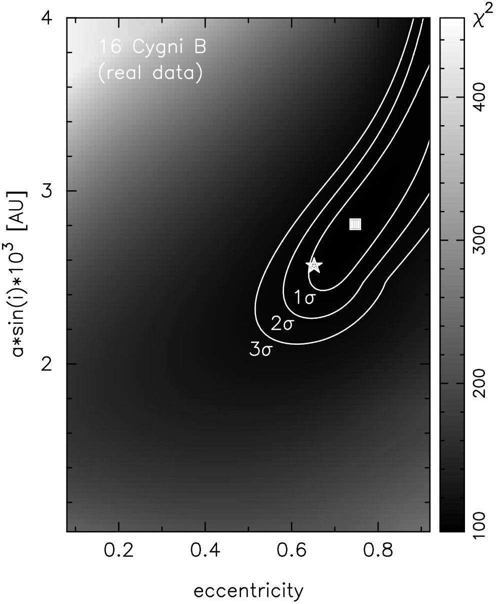

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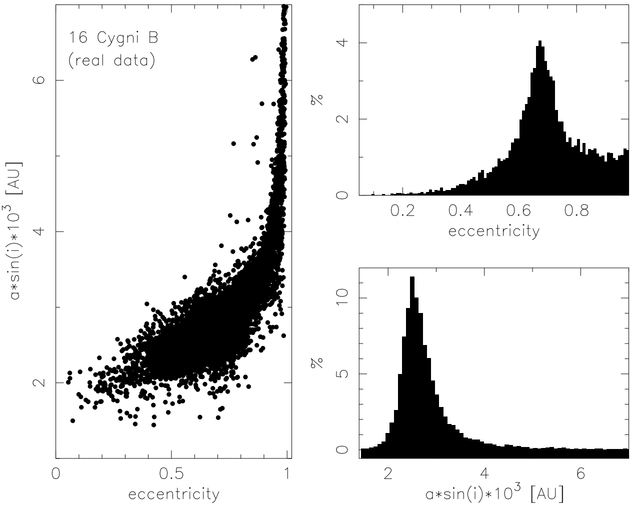
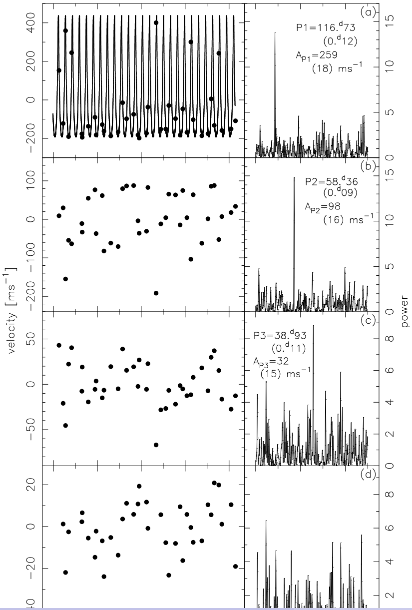


A planet around 70 Virginis.



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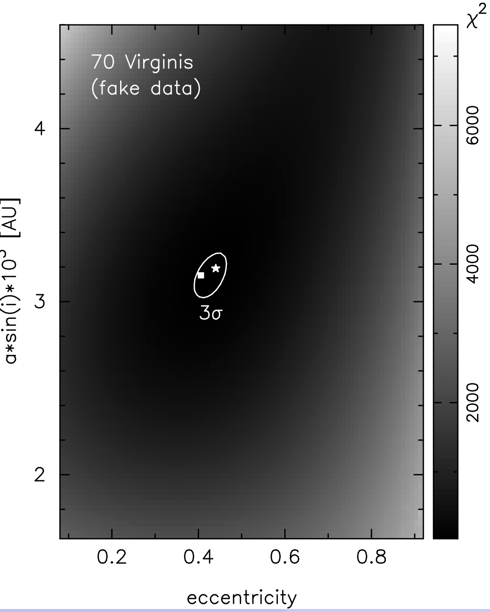

Serious problems

- By the way, what is the number of planets. (Ockham's razor)
- Do all good fits are equally good?
- Can we predict parameters of an additional planet from a linear trend.
- Mass determinations from rv observations.

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The stability razor

The most important observable: the fact that the planetary system exists!

The true problem is: find $\boldsymbol{p}_0$ such that

$$\chi^2(\boldsymbol{p}_0) = \min_{\boldsymbol{p} \in S} \chi^2(\boldsymbol{p})$$

$$S \subset \mathbb{R}^k,$$

S is a subset of the parameters space corresponding to stable systems

Two approches

1. First fit then check and correct.
2. Check while fitting.

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How to measure stability?

Lyapunov exponent

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} = \boldsymbol{v}(\boldsymbol{x}), \qquad \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\delta} = \frac{\partial \boldsymbol{v}}{\partial \boldsymbol{x}}(\boldsymbol{x})\boldsymbol{\delta}, \qquad \boldsymbol{x}, \boldsymbol{\delta} \in \mathbb{R}^n,$$

$$\boldsymbol{x}(t_0) = \boldsymbol{x}_0, \qquad \boldsymbol{\delta}(t_0) = \boldsymbol{\delta}_0 \neq \boldsymbol{0},$$

Definition

$$\lambda = \lambda(\boldsymbol{x}_0, \boldsymbol{\delta}_0) = \lim_{t\to\infty} \frac{1}{t} \ln \frac{\|\boldsymbol{\delta}(t)\|}{\|\boldsymbol{\delta}_0\|}$$

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Mean Exponential Growth of Nearby Orbits
Cincotta, P. and C. Simó, 2000, ApJ Sup. Ser., 147, 205

$$\lambda := \lim_{t\to\infty} \frac{1}{t}\int_0^t \frac{\dot{\delta}(s)}{\delta(s)}\,\mathrm{d}s = \left\langle \frac{\dot{\delta}}{\delta} \right\rangle,$$

$$\delta = \|\boldsymbol{\delta}\|, \qquad \dot{\delta} = \frac{\mathrm{d}}{\mathrm{d}t}\delta = \frac{\dot{\boldsymbol{\delta}}\cdot\boldsymbol{\delta}}{\|\boldsymbol{\delta}\|^2}.$$

$$Y(t) := \frac{2}{t}\int_0^t \frac{\dot{\delta}(s)}{\delta(s)} s\,\mathrm{d}s, \qquad \langle Y\rangle(t) := \frac{1}{t}\int_0^t Y(s)\,\mathrm{d}$$

MEGNO properties

1. If $\boldsymbol{x}(t, \boldsymbol{x}_0)$ is regular then
$$\lim_{t\to\infty} \langle Y \rangle(t) = 2.$$
2. If $\boldsymbol{x}(t, \boldsymbol{x}_0)$ ic chaotic then $\langle Y \rangle(t) \approx (\lambda/2)t$ as $t \to \infty$.

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MEGNO calculations

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} = \boldsymbol{v}(\boldsymbol{x}), \qquad \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\delta} = \frac{\partial \boldsymbol{v}}{\partial \boldsymbol{x}}(\boldsymbol{x})\boldsymbol{\delta}, \qquad \boldsymbol{x}, \boldsymbol{\delta} \in \mathbb{R}^n,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}y = \frac{\dot{\boldsymbol{\delta}} \cdot \boldsymbol{\delta}}{\boldsymbol{\delta} \cdot \boldsymbol{\delta}} t, \qquad \frac{\mathrm{d}}{\mathrm{d}t}w = 2\frac{y}{t}.$$

Then $Y(t) = 2y(t)/t$ and $\langle Y \rangle(t) = w(t)/t$.

Other even better methods

Spectral Number

Michtchenko, T. and Ferraz-Mello, S. 2001, ApJ, 122, 474.

GAMP

Genetic Algorithm with MEGNO Penalty

1. Genetic algorithm, Charbonneau, P. 1995, ApJS, 101, 309.
   `http://www.hao.ucar.edu/public/research/si/pikaia/pikaia.html`
2. MEGNO (or SN, or any other) penalty.
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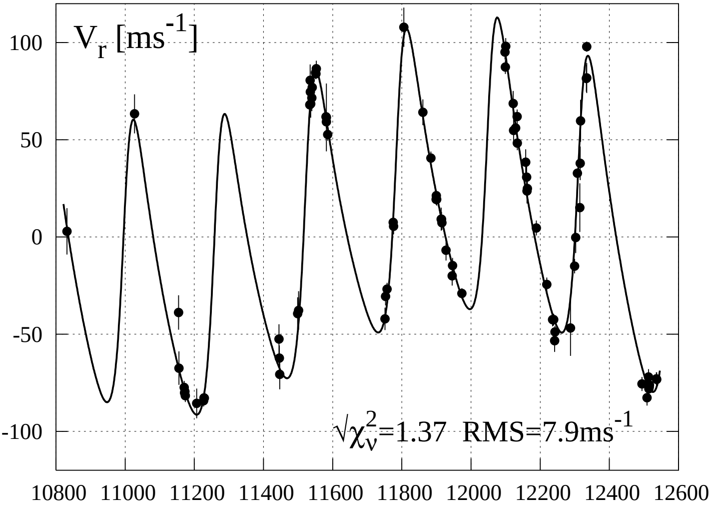
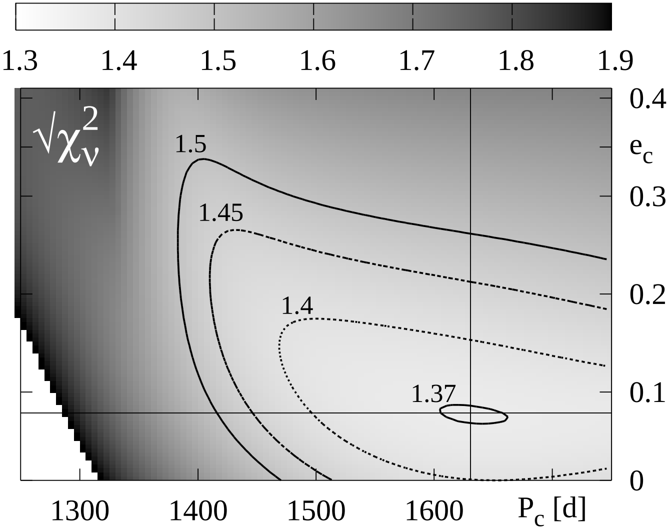
Observations

86 points; $\sigma \approx 3 - 6\mathrm{m/s}$ for Keck; $\sigma \approx 7 - 17\mathrm{m/s}$ for Lick.

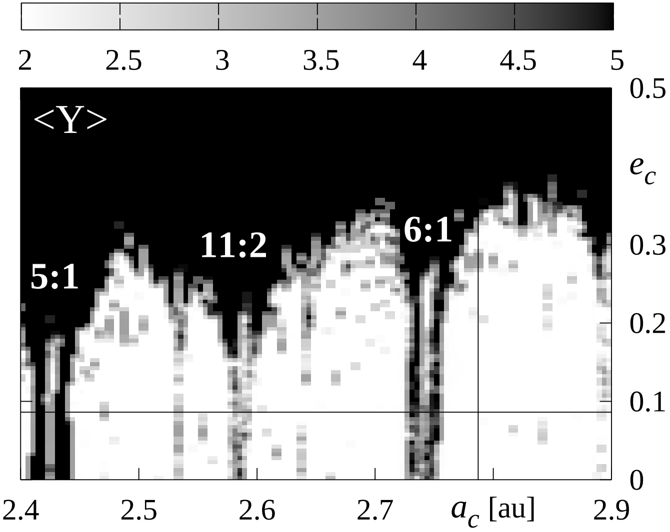


χ map



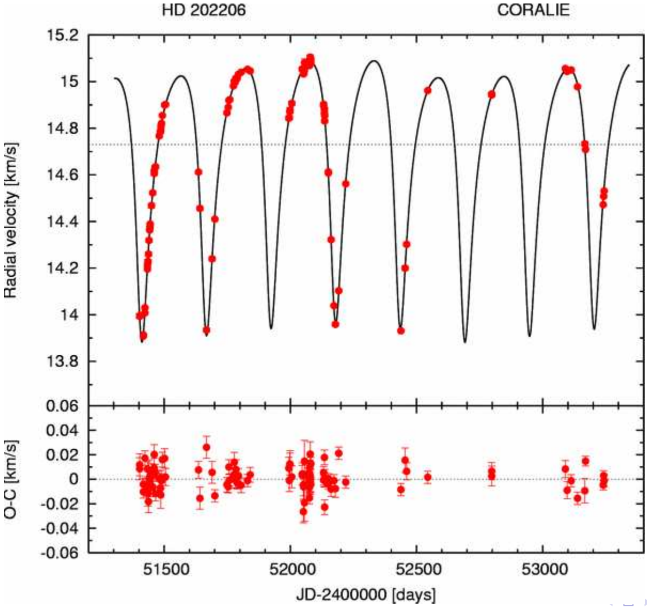
1.3
1.4
1.5
1.6
1.7
1.8
1.9
√χ²ν
1.5
1.45
1.4
1.37
0.4
ec
0.3
0.2
0.1
0
1300
1400
1500
1600
Pc [d]

Stability



Outline

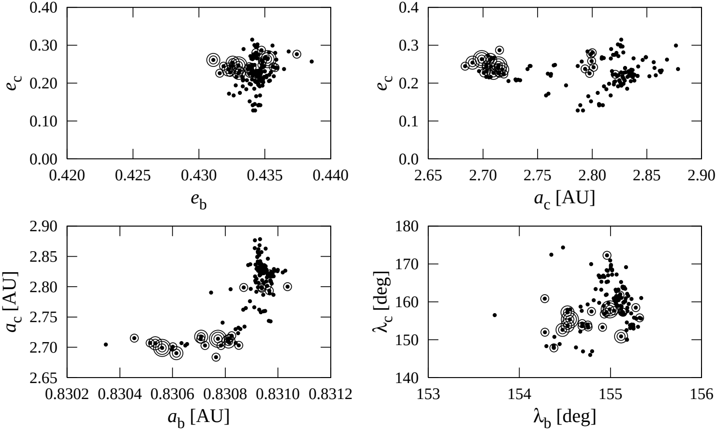
RV curve


HD 202206
CORALIE
Radial velocity [km/s]
O-C [km/s]
15.2
15
14.8
14.6
14.4
14.2
14
13.8
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
51500
52000
52500
53000
JD-2400000 [days]

Estimated parameters

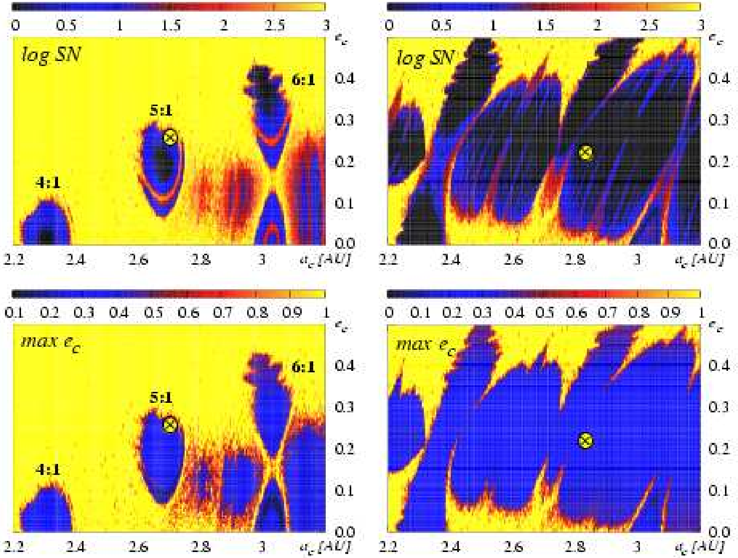
- $\mathrm{M_b} = 17.5\mathrm{M_J}$, $\mathrm{M_c} = 2.4\mathrm{M_J}$.
- $\mathrm{P_b} = 256\mathrm{d}$, $\mathrm{P_c} = 1297, 1383\mathrm{d}$.
- $e_\mathrm{b} = 0.43$, $e_\mathrm{c} = 0.27$.
- $5 : 1$ MMR?
- Keplerian and Newtonian fits give rise desintegrating system ($\approx 10^3$ y)

GAMP results

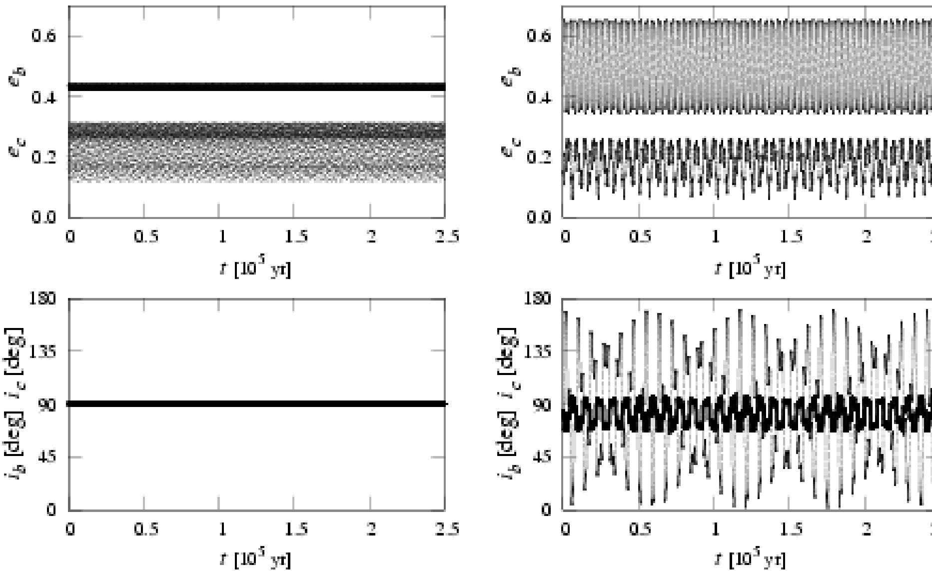


GAMP results

left $\chi_\nu = 1.53$, rms $= 9.97$ m/s; right $\chi_\nu = 1.62$, rms $= 10.32$ m/s;

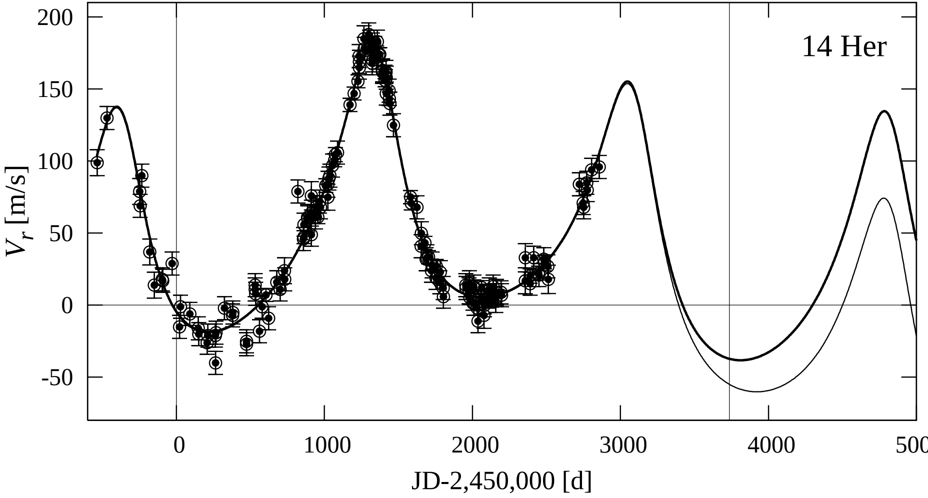


Coplanar or not?



Outline

RV curve

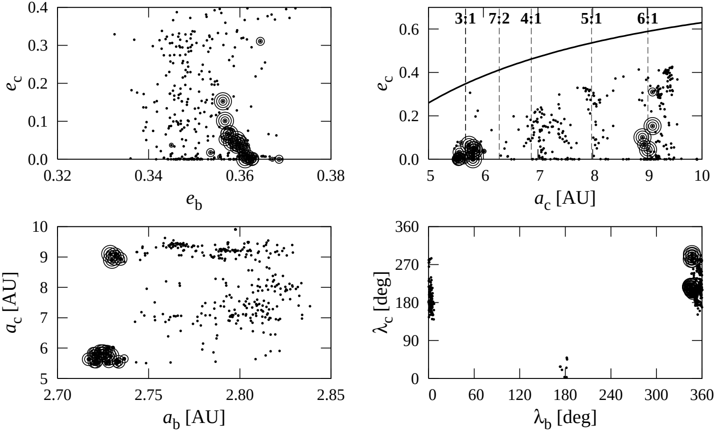


14 Her
200
150
100
50
0
-50
V_r [m/s]
0
1000
2000
3000
4000
JD-2,450,000 [d]

Problem

- $\mathrm{M_b} = 4.64\mathrm{M_J}$, $e_\mathrm{b} = 0.34$, $\mathrm{P_b} = 1773\mathrm{d}$ and linear trend 3.6m/s per year.
- best Keplerian+drift fits: rms= 11m/s but mean error of observations $\sigma_\mathrm{m} = 7.2$m/s.

GAMP results



New updated analysis

K. Gozdziewski, C. Migaszewski, M. Konacki, arXiv:0705.1858

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Problem

- 2001 Butler et al., $P_b \approx 630$d.
- 2002 Jones et al., a linear trend.
- 2004 McCarthy et al. , $P_c \approx 3000$d, $e_c = 0.57$.
- 2004 Santos et al., $P_d \approx 9$d, $M_d = 0.044 M_J$.
- What about panet HD 160691e?

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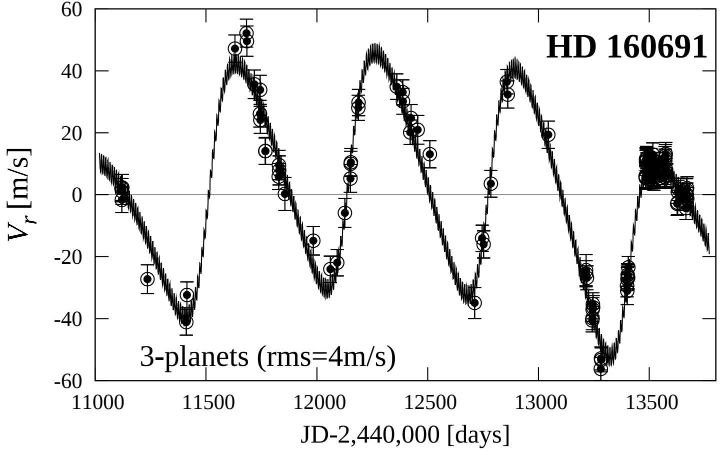
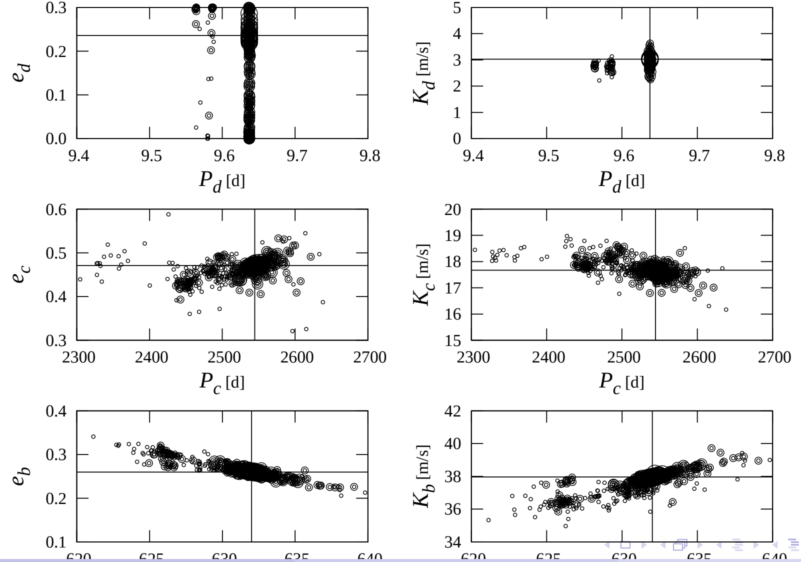

HD 160691
3-planets (rms=4m/s)
V_r [m/s]
JD-2,440,000 [days]
60
40
20
0
-20
-40
-60
11000
11500
12000
12500
13000
13500

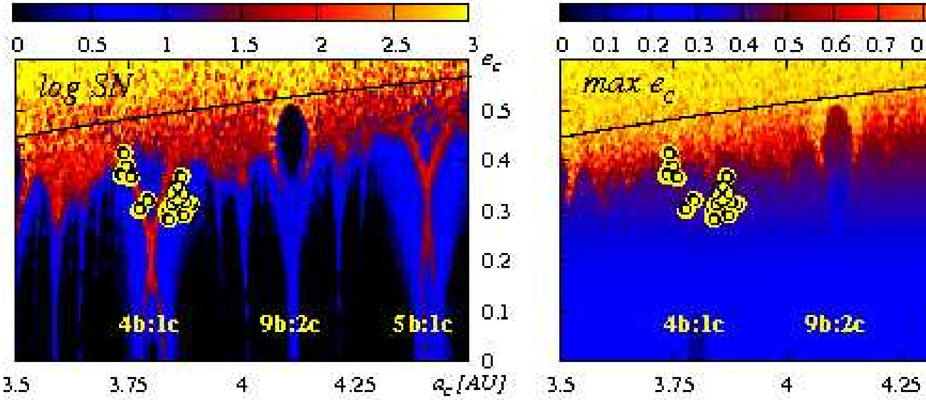
3 Keplerian planets+ GA

$e_c = 0.47$ strong unstable solutions.

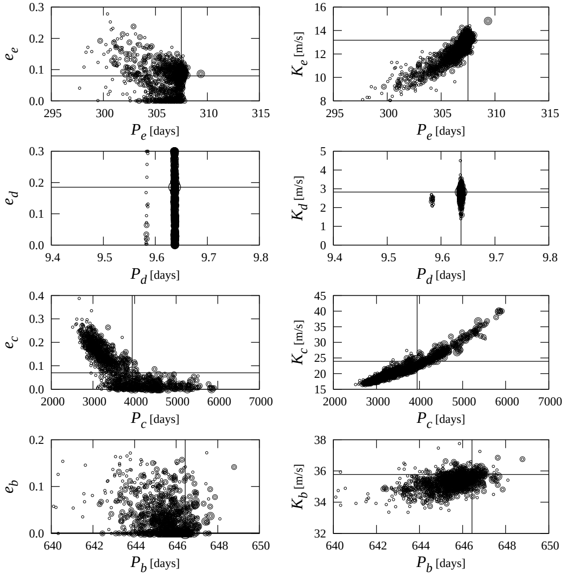


2(3) planets+ GAMP

2004 McCarthy et al. observations: for stable solutions $a_c > 4$AU.



4 Keplerian planets+ GA



stable solutions

Much more in

1. Goździwski, K. et. al. 2007, ApJ, 657, pp. 546-558.
2. Pepe, F. et. al. 2007, A& A, 426, pp. 769-776; different observations and similar conclusions.

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Assumptions of simulations

1. 2 planets.
2. 3D Newtonian model.
3. Parametrisation: Keplerian elements, initial conditions;
4. t_i random.
5. $\sigma_i = \sigma = 3$m/s.
6. GA+ML

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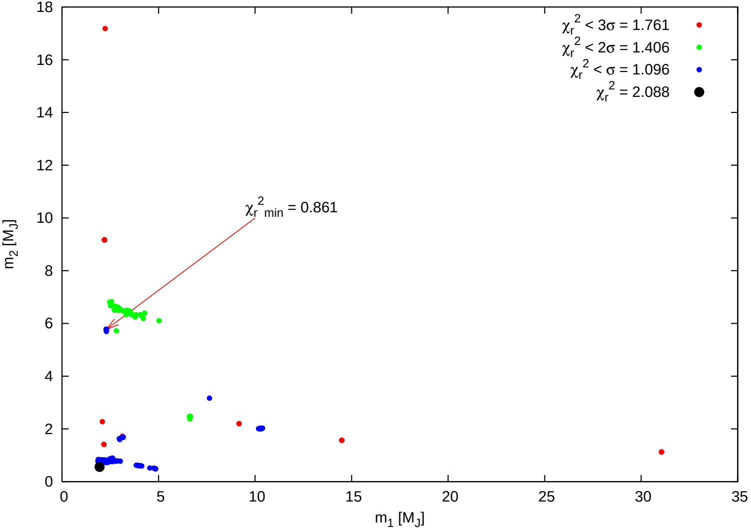
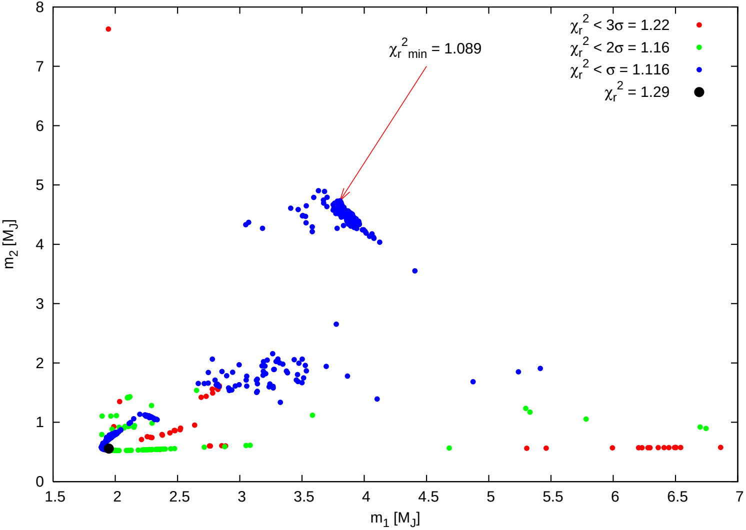
2:1 MMR, $2P_2$

$N_{\text{obs}} = 20$



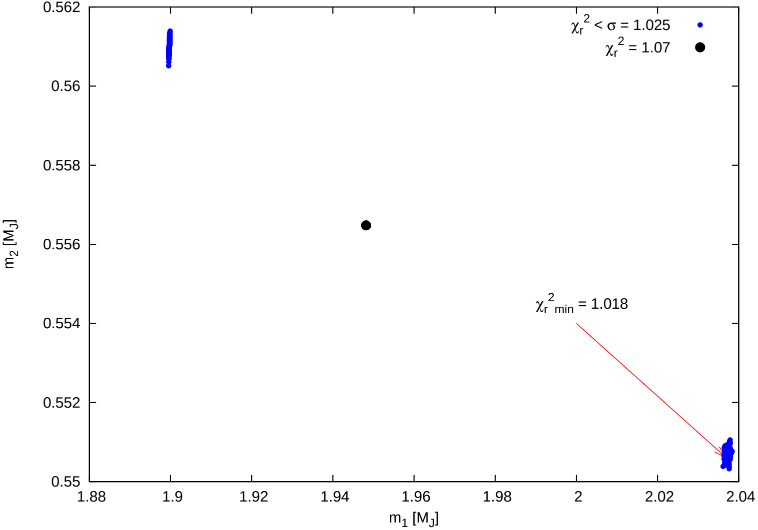
2:1 MMR, $5P_2$

$N_{\rm obs} = 54$



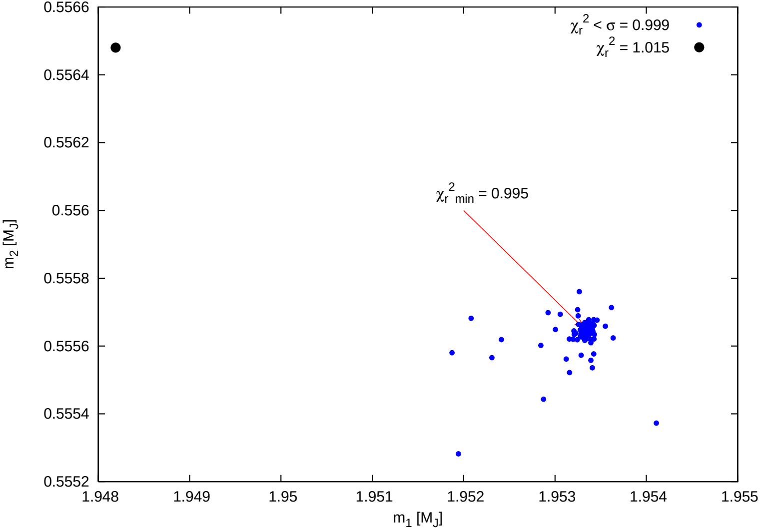
2:1 MMR, $15P_2$

$N_{\rm obs} = 161$



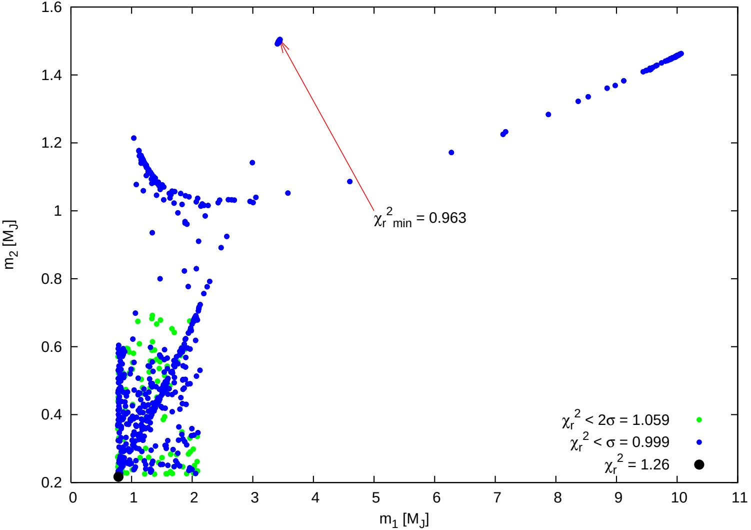
2:1 MMR, $30P_2$

$N_{\mathrm{obs}} = 325$



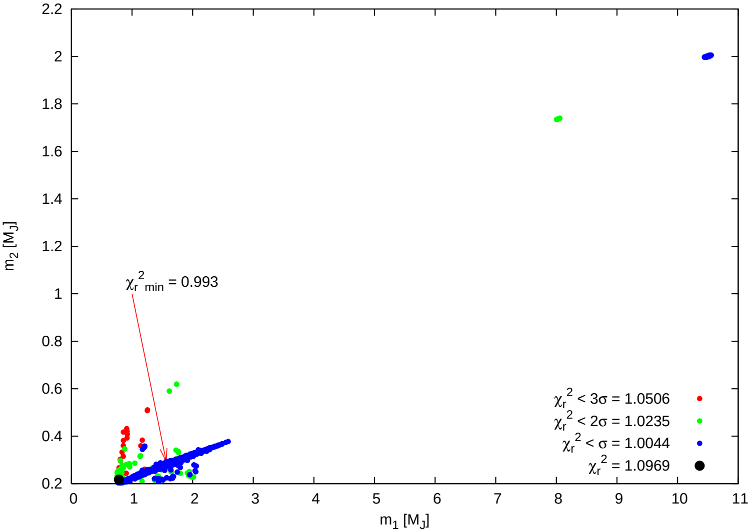
3:1 MMR, $2P_2$

$N_{\mathrm{obs}} = 47$



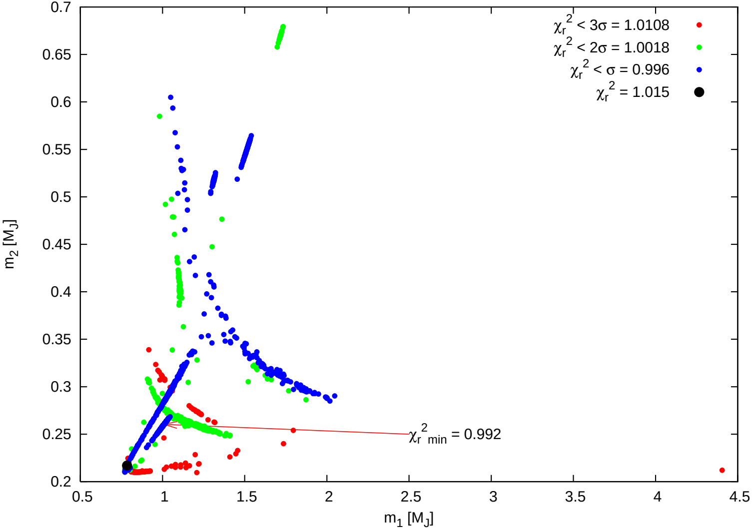
3:1 MMR, $5P_2$

$N_{\mathrm{obs}} = 115$



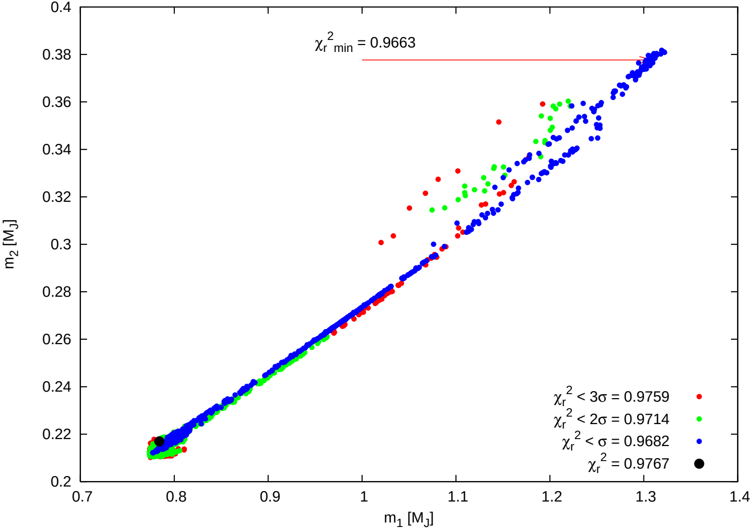
3:1 MMR, $15P_2$

$N_{\text{obs}} = 325$



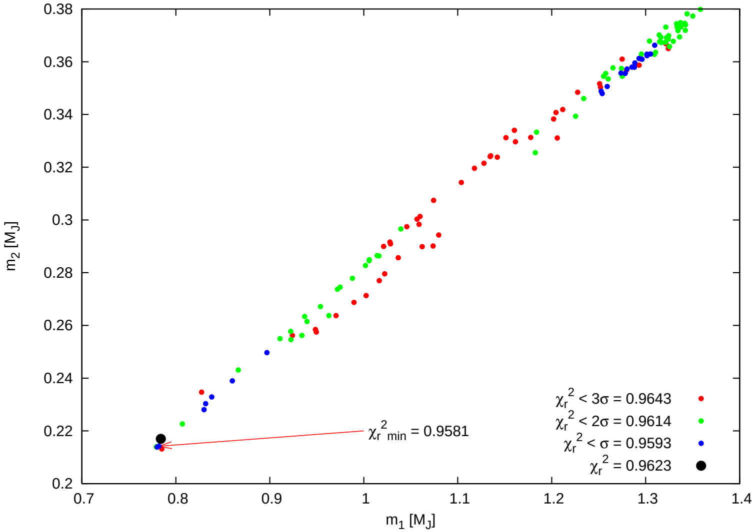
3:1 MMR, $30P_2$

$N_{\text{obs}} = 650$



3:1 MMR, $100P_2$

$N_{\text{obs}} = 1000$



References

1. God́ziewski, Krzysztof; Maciejewski, Andrzej J.; Migaszewski, Cezary, ApJ, 651, pp. 546-558.
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