Orbit determination for extra-solar planetary systems.
Lecture I

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Outline

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1. Extra-solar planets
2. Observations
3. Modeling observations
4. Fitting orbits.

Early and false discoveries

- The Barnard star planetary system.
- Peter van de Kamp observations 1938–1982 (about 10000 plates).
- Final estimate: two planets with masses: $M_1 = 0.7M_\mathrm{J}$ and $M_2 = 0.5M_\mathrm{J}$ and periods $P_1 = 12$y and $P_2 = 20$y, respectively; (Vistas in Astronomy, 26:141–157, 1982).

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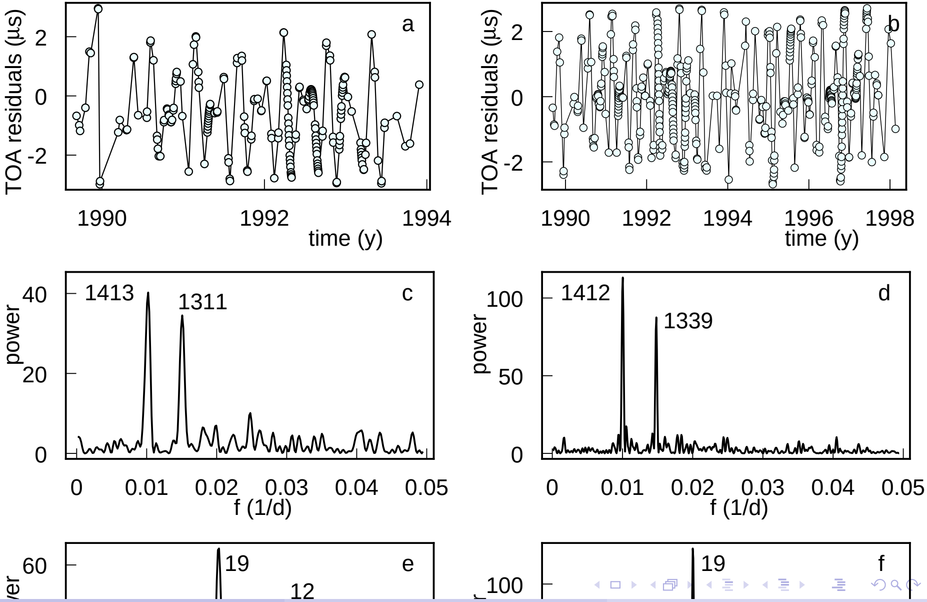

51Peg planet

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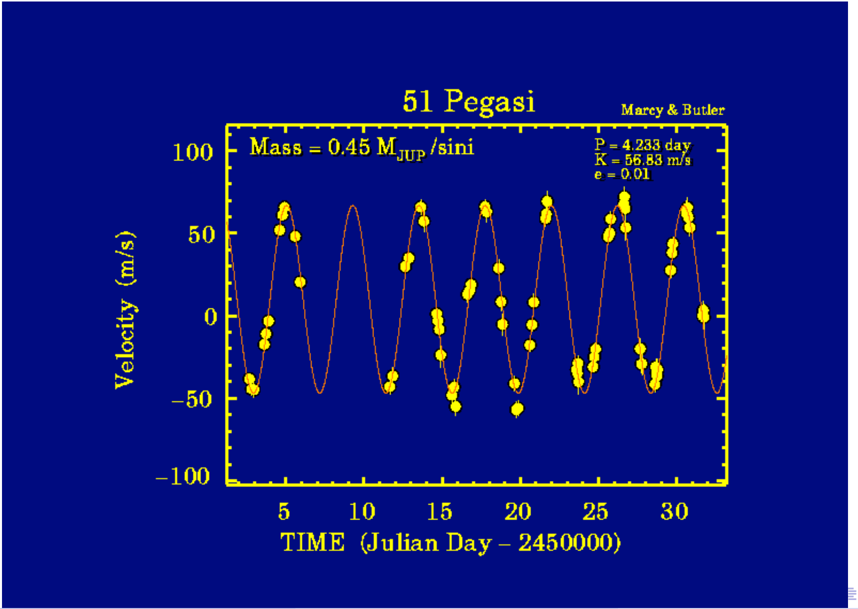

Actually (24 May, 2007)

- Radial velocities: 224 planets, 23 multiple planetary systems.
- Transit: 20 planets.
- Microlensing: 4 planets.
- Imaging: 4 planets.
- Timing: 4 planets, 1 multiple planetary system.
- All: 236 planets, 24 multiple planetary systems.

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- SIM.
- Kepler.
- . . .

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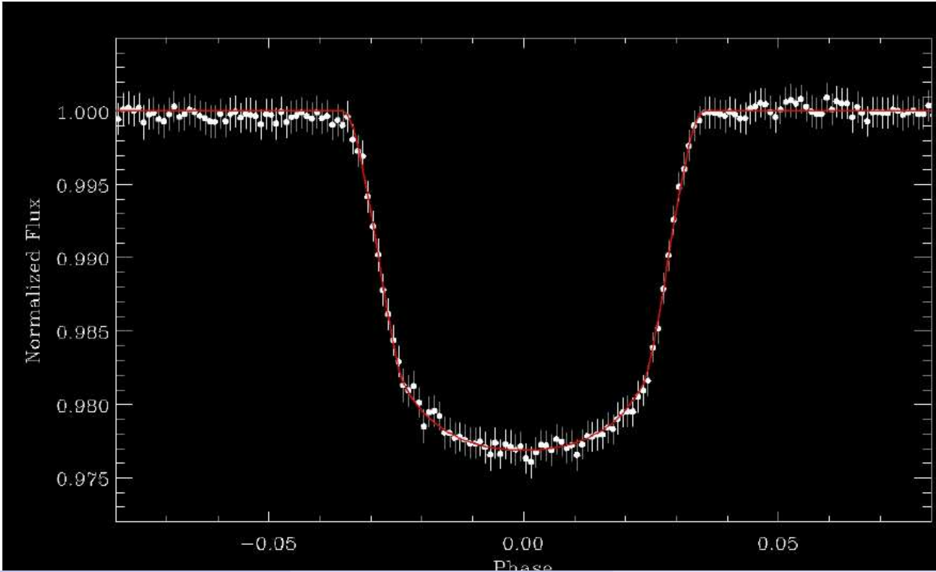
Future

Tomorrow—it is today but tomorrow. —St. Mrożek



Outline

Detection methods

- Radial velocities.
- Pulsars timing.
- Transits.
- Imaging.
- Microlensing.
- Optical interferometry.

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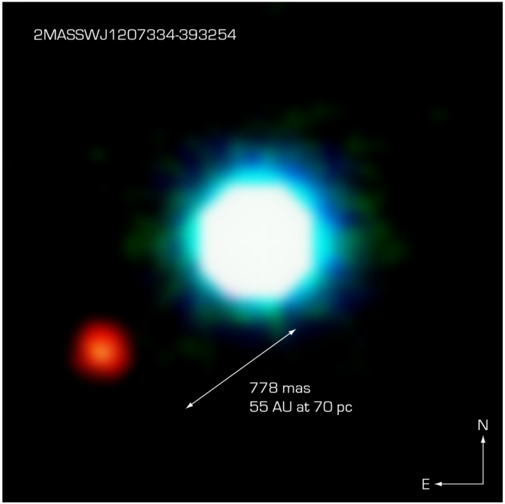
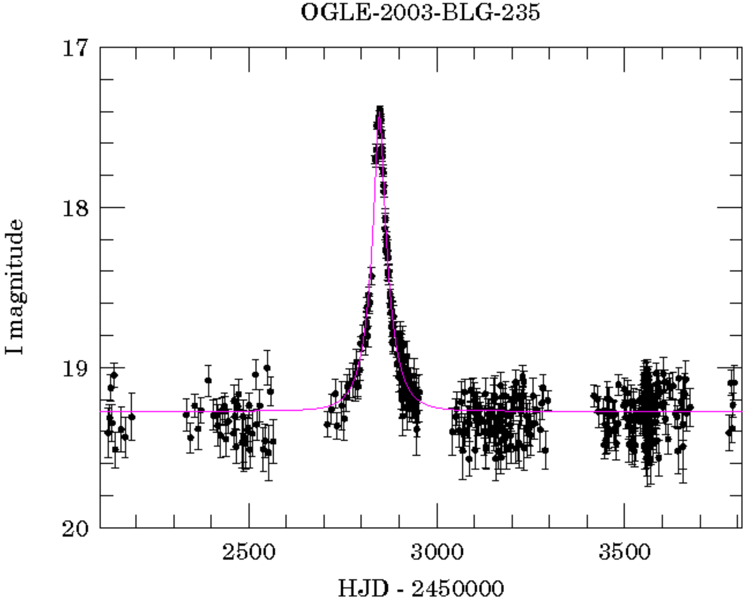
Imaging



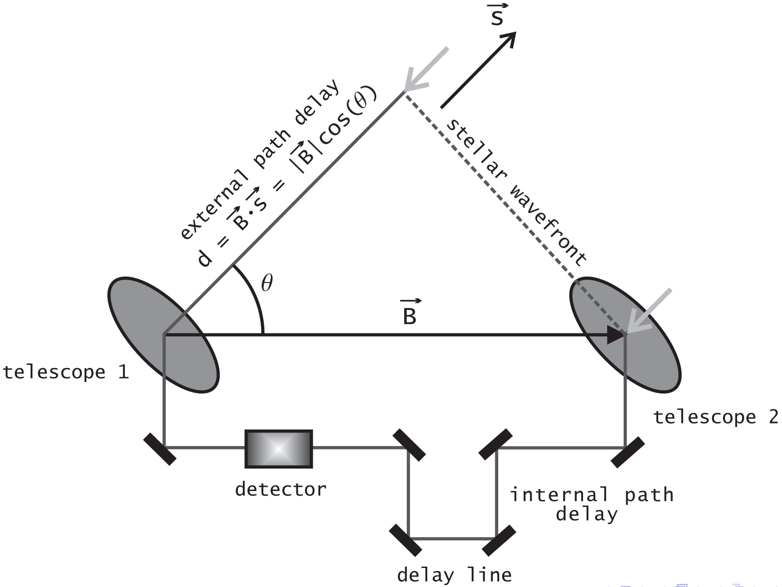
NACO Image of the Brown Dwarf Object 2M1207 and GPCC

ESO PR Photo 26a/04 (10 September 2004)

Microlensing

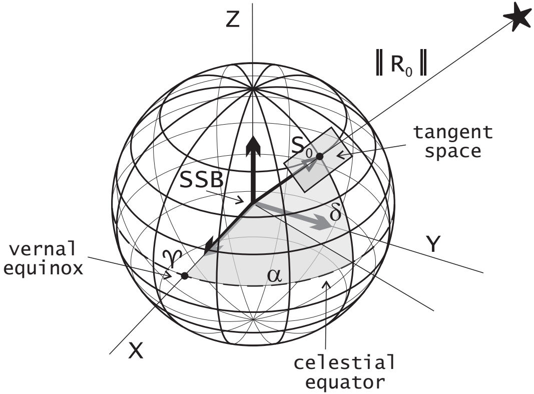


Space interferometry



Outline

Geometry


Z
‖ R0 ‖
tangent
space
S0
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δ
Y
vernal
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ϒ
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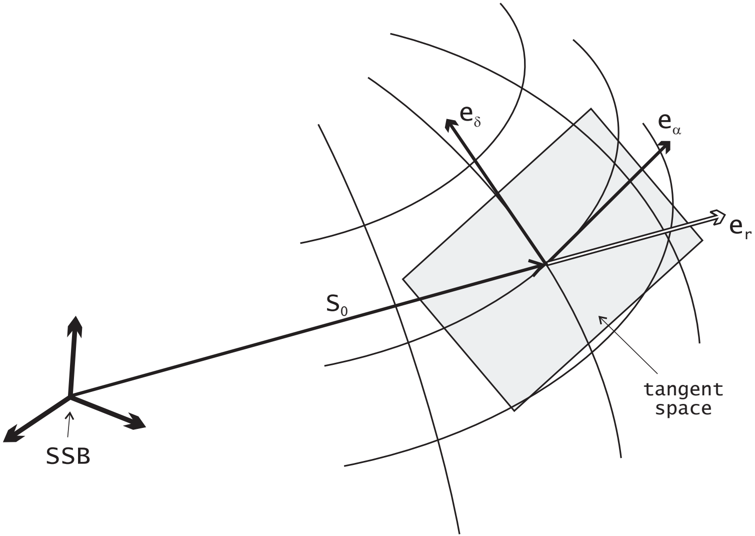

Basic idea

If $\boldsymbol{V}^{\star}(t)$ is the barycentric velocity of the star and $\boldsymbol{e}_{\mathrm{r}}$ the unit vector from observer to the star planetary system then the observed radial velocity is given by

$$v_{\mathrm{r}}(t) := \boldsymbol{e}_{\mathrm{r}} \cdot \boldsymbol{V}^{\star}(t).$$

Thus, if $\boldsymbol{e}_3 = \boldsymbol{e}_{\mathrm{r}}$, then $v_{\mathrm{r}}(t) = V_3^{\star}$, and $v_{\mathrm{r}}(t)$ does not depend how we choose $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$.

One planet.

The mass center

$$m\boldsymbol{R} + m_\star \boldsymbol{R}^\star = \boldsymbol{0}, \qquad m\boldsymbol{V} + m_\star \boldsymbol{V}^\star = \boldsymbol{0}$$

$$\boldsymbol{r} = \boldsymbol{R} - \boldsymbol{R}^\star, \qquad \boldsymbol{v} = \boldsymbol{V} - \boldsymbol{V}^\star,$$

$$\boldsymbol{R}^\star = -\frac{m}{m_\star}\boldsymbol{R} = -\frac{m}{m_\star + m}\boldsymbol{r}$$

$$\boldsymbol{V}^\star = -\frac{m}{m_\star}\boldsymbol{V} = -\frac{m}{m_\star + m}\boldsymbol{v}$$

$$\ddot{\boldsymbol{R}} = -\frac{\mu_{\mathrm{r}}}{R^3}\boldsymbol{R}, \qquad \mu_{\mathrm{b}} = \frac{Gm_\star^3}{(m_\star + m)^2}$$

$$\ddot{\boldsymbol{r}} = -\frac{\mu_{\mathrm{r}}}{r^3}\boldsymbol{r}, \qquad \mu_{\mathrm{r}} = G(m_\star + m),$$

Kepler orbit

$$\boldsymbol{R}(t) = \frac{a(1-e^2)}{1+e\cos(\nu)}\left[\cos(\nu)\boldsymbol{P} + \sin(\nu)\boldsymbol{Q}\right],$$

$$\boldsymbol{V}(t) = \frac{an}{\sqrt{1-e^2}}\left[-\sin(\nu)\boldsymbol{P} + (e+\cos(\nu))\boldsymbol{Q}\right],$$

$$\boldsymbol{P} = \boldsymbol{l}\cos(\omega) + \boldsymbol{m}\sin(\omega), \quad \boldsymbol{Q} = -\boldsymbol{l}\sin(\omega) + \boldsymbol{m}\cos(\omega),$$

$$\boldsymbol{l} = \begin{bmatrix}\cos\Omega\\ \sin\Omega\\ 0\end{bmatrix}, \quad \boldsymbol{m} = \begin{bmatrix}-\cos i\sin\Omega\\ \cos i\cos\Omega\\ \sin i\end{bmatrix},$$

$$\tan\frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}, \quad E - e\sin E = n(t - T_{\mathrm{P}}), \quad n^2a^3 = \mu.$$

Final formula

$$v_{\mathrm{r}}(t) = -K\left[\cos(\nu(t)+\omega) + e\cos\omega\right] + v_0,$$

$$K = \frac{\sigma a n \sin i}{\sqrt{1-e^2}}, \qquad \sigma = \frac{m}{m_\star}.$$

Parameters

$$\boldsymbol{p} = (K, n, e, \omega, T_{\mathrm{p}}, v_0)$$

More than one planet.

- Keplerian models.
- Garvitational N-body problem.

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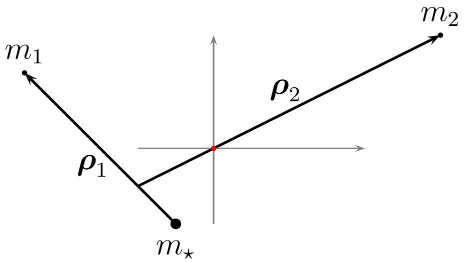
More than one planet. Keplerian models.

Just add

$$v_{\mathrm{r}}(i) = \sum_{k=1}^{n} v_{\mathrm{r}}^{(k)} + v_0,$$

$$v_{\mathrm{r}}^{(k)} = -K_k \left[\cos(\nu_k(t) + \omega_k) + e_k \cos \omega_k\right], \quad k = 1, \dots, n.$$

Yet another Keplerian model.



Jacobi coordinates

$$v_{\mathrm{r}} = -K_1 f_1(t) - K_2 f_2(t),$$

$$K_k = \frac{\sigma_k a_k n_k \sin i_k}{\sqrt{1-e_k^2}}, \quad f_k(t) = \cos(\nu_k(t) + \omega_k) + e_k \cos\omega_k$$

$$\sigma_1 = \frac{m_1}{m_\star + m_1}, \quad \sigma_2 = \frac{m_2}{m_\star + m_1 + m_2}$$

$$\mu_1 = G(m_\star + m_1), \quad \mu_1 = G(m_\star + m_1 + m_2)$$

Gravitational N-body model.

Barycentric reference frame

$$\left.\begin{aligned} \dot{\boldsymbol{R}}_i &= \boldsymbol{V}_i, \\ m_i\dot{\boldsymbol{V}}_i &= -\sum_{j=0}^{n}{}' \frac{Gm_im_j}{R_{ij}^3}(\boldsymbol{R}_i - \boldsymbol{R}_j), \quad \boldsymbol{V}_i := \dot{\boldsymbol{R}}_i \end{aligned}\right\}, \quad i = 0, \ldots, n,$$

$$\sum_{i=0}^{n} m_i\boldsymbol{R}_i = \boldsymbol{0}, \qquad \sum_{i=0}^{n} m_i\dot{\boldsymbol{V}}_i = \boldsymbol{0}$$

$$v_{\mathrm{r}}(t) = -\sum_{i=1}^{n} \frac{m_i}{m_0} V_{i,3}.$$

Gravitational N-body model. Parametrisation.

1. Osculating elements: $(m_k, a_k, e_k, i_k, \omega_k, \Omega_k, T_{\mathrm{p},k})$, for $k = 1, \ldots, n$, and $\Omega_1 = 0$!
2. Initial condtition: $(m_k, \boldsymbol{R}_k(t_0), \boldsymbol{V}_k(t_0))$, for $k = 1, \ldots, n$, and fix e.g. $R_{1,1} = 0$!

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An opinion.

An astrophysicist can fit a sinusoid to two observations and moreover he claims that this fit is very good.

— Ryszard Szczerba, astrophysicist

Classical χ^2 method.

One-dimensional observations $x(t)$.

- Observations: $x_1, \ldots, x_N$, $x_i = x(t_i)$
- Errors of individual observations: $\sigma_1, \ldots, \sigma_N$.
- Model: $\hat{x} = \hat{x}(t, \boldsymbol{p})$, $\boldsymbol{p} = (p_1, \ldots, p_k) \in \mathbb{R}^k$.
- $$\chi^2(\boldsymbol{p}) = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{x_i - \hat{x}(t_i, \boldsymbol{p})}{\sigma_i} \right)^2,$$
- degrees of freedom: $\boldsymbol{u} = N - k - 1$,

$$\chi^2_\nu(\boldsymbol{p}) = \frac{1}{\nu} \sum_{i=1}^{N} \left(\frac{x_i - \hat{x}(t_i, \boldsymbol{p})}{\sigma_i} \right)^2.$$

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Classical χ^2 method.

Problem: find $\boldsymbol{p}_0 \in \mathbb{R}^k$ such that

$$\chi^2(\boldsymbol{p}_0) = \min_{p \in \mathbb{R}^k} \chi^2(\boldsymbol{p}).$$

How to find the global minimum?

Find all local minma and choose the smalest one!

Facile dictu dificile factu!

Local minimum

$$\boldsymbol{p}_0 = \min_{\boldsymbol{p}\in U} F(\boldsymbol{p}), \qquad U \in \mathbb{R}^k;$$

Condition: $\|\boldsymbol{p} - \boldsymbol{p}_0\|$ is small. Then

$$F(\boldsymbol{p}) = F(\boldsymbol{p}_0) + \nabla F(\boldsymbol{p}_0)\boldsymbol{x} + \frac{1}{2}\boldsymbol{x}^T \nabla^2 F(\boldsymbol{p}_0)\boldsymbol{x} + \cdots ,$$

$$\boldsymbol{x} = \boldsymbol{p} - \boldsymbol{p}_0, \quad A := \left[\frac{\partial^2 F}{\partial p_i \partial p_j}(\boldsymbol{p}_0)\right].$$

Quadratic approximation

1. $\boldsymbol{p}_1$ close to $\boldsymbol{p}_0$;
2. $\boldsymbol{p}_2 = \boldsymbol{p}_1 - A^{-1}\boldsymbol{y}$; $\boldsymbol{y} = \nabla F(\boldsymbol{p}_1)$, $A = \nabla^2 F(\boldsymbol{p}_1)$.
3. $\|\boldsymbol{p}_1 - \boldsymbol{p}_2\| < \epsilon$?
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Problems

- Expensive calculations of $\nabla F(\boldsymbol{p})$ and/or $\nabla^2 F(\boldsymbol{p})$. Solution: choose the simplex method.
- Initial approximation. Monte Carlo, periodograms, or pseudo-global genetic algorithm.
- Natural constrains.