Magnetic Flux Cancellation and Coronal Magnetic Energy

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ABSTRACT

I investigate the processes at work in the cancellation of normal magnetic flux in solar magnetograms, and study the relationships between cancellation and the budget of free energy in the coronal magnetic field that can power solar flares and CMEs. I begin by defining cancellation, and present the equations describing the evolution of free energy during cancellation. I then analyze these equations, to show that: cancellation tends to reduce the magnetic energy of the hypothetical "open" field state; in a sheared arcade field, steady cancellation can add free energy to the actual coronal field; and, considering relevant boundary conditions, the horizontal magnetic flux density can increase while normal flux is cancelling, even in the presence of constant resistivity. Finally, I discuss the implications of these facts.

1. Introduction and Definitions

Observers who first saw magnetic flux cancellation in magnetograms of the solar photosphere offered an operational definition of the phenomenon, "the mutual apparent loss of magnetic flux in closely-spaced features of opposite polarity" (Livi *et al.* 1985).

Physically, Zwaan (1987) offered three scenarios to explain this phenomenon in terms of magnetic reconnection, illustrated schematically in Figure 1. The pictures vary essentially only in the difference in altitude between the (idealized) layer in which the magnetograph images the cancelling magnetic fluxes, and that at which the reconnection between the converging magnetic flux systems occurs. Zwaan's "reconnective cancellation" scenarios are consistent with the observations of Harvey *et al.* (1999) that, in the quiet sun, cancelling magnetogram features typically originate in distinct flux systems and coincide with coronal bright points. More recently, observers have reported detailed signatures consistent with the magnetic reconnection implied by this process, assuming it takes place above the magnetograph imaging layer; see, e.g. Chae (2003). Other observers attribute the disappearance of vertical flux to the essentially ideal submergence of inverted-U-shaped field lines (Chae *et al.* 2004; Harvey *et al.* 1999; Rabin *et al.* 1984) or emergence of U-shaped field lines (van Driel-Gesztelyi *et al.* 2000). Harvey *et al.* (1999) argued that emerging U-shaped loops were far less common (if present at all) than submerging, inverted-U loops in their sample of reconnective cancellations. It is likely that each of these processes are at work on the Sun in some cases, either singly and in combination.

Martin (1998) considers cancellation a necessary condition for the formation of prominences, which are extended coronal emission features, seen in chromospheric spectral lines (e.g., H-α) when over the limb. When on the disk, these structures are visible as absorption features and are known as filaments. The sudden eruption of prominences, sometimes occurring over a few tens of coronal Alfvén crossing times, $\tau_{\rm cor}$, can lead to coronal mass ejections, or CMEs (Forbes 2000), the primary drivers of severe space weather disturbances (Gosling 1993).

An open question is how the coronal magnetic energy (from now on, "energy" refers to magnetic energy, unless explicity stated otherwise), $U_M = \int_V dV (\mathbf{B}\cdot\mathbf{B})/8\pi$, evolves when flux cancels during prominence formation. In addition to affecting the energy in the the actual magnetic field, $\mathbf{B}$, cancellation also affects the energy of the potential magnetic field, $\mathbf{B}^{(P)}$, that matches the normal field boundary condition on $\mathbf{B}$, i.e., $B_n|_{\partial V} = B_n^{(P)}|_{\partial V}$, where ∂V is the surface that bounds V. The potential field is current free, i.e., $4\pi\mathbf{J}^{(P)} = c(\nabla \times \mathbf{B}^{(P)}) = 0$, implying $\mathbf{B}^{(P)}$ can be expressed as the gradient of a scalar potential, $\mathbf{B}^{(P)} = -\nabla\chi$. The potential field is of interest because it is the *unique, minimum energy* field that matches the same normal field boundary condition as $\mathbf{B}$. Since the photosphere remains essentially unchanged on the rapid timescale of dynamic coronal field evolution ($\tau_{\rm p-sph} \gg \tau_{\rm cor}$), the energy available to drive such evolution is the free magnetic energy, $U_F = U_M - U_M^{(P)}$, where $U_M^{(P)} = \int_V dV (\mathbf{B}^{(P)} \cdot \mathbf{B}^{(P)})$.

The coronal field can effectively store free magnetic energy because its evolution is constrained by its fixed topology: the corona's long length scales, high temperature, and low density mean that the field's evolution is, normally, ideal, or nearly so; equivalently, the magnetic Reynolds number is large. Consequently, while the coronal plasma possesses sufficient degrees of freedom to always reside in a minimum *accessible* energy state, the potential state is not always accessible. In the general case, $\mathbf{B}$ cannot relax to $\mathbf{B}^{(P)}$, the global energy minimum among the set of all magnetic fields that match the normal field boundary condition, $B_n|_{\partial V}$, but instead relaxes to a local energy minimum. Given the dominance of the Lorentz force over other terms in the momentum equation that describes the coronal field's evolution, the accessible minimum state, consistent with the field's topology, is presumably force-free, $(\nabla \times \mathbf{B}) \times \mathbf{B} = 0$.

Even when the field's topology changes rapidly, in a process known as fast magnetic reconnection (perhaps due to the relatively brief occurrence of a spatially localized enhancement of magnetic diffusivity, perhaps manifested as a flare or sub-flare), the field is *still* unable to relax to the potential state, because the field's magnetic helicity (Berger and Field 1984) is approximately conserved (Berger 1984). (By definition, fields that possess gauge-invariant magnetic helicity are not potential.) Instead, Taylor (1985), has suggested that the field relaxes to a linear, "constant-α" force-free state with approximately the same magnetic helicity as the field possessed prior to the onset of reconnection. Antiochos *et al.* . (2002) have argued that localized nature of reconnection in the solar case (in contrast to the global relaxation in the terrestrial plasma experiments Taylor characterized), results in a variable-α (or non-linear) force-free field. Regardless of whether α varies in space or is constant, the presence of helicity in the post-reconnection field implies that the field is not potential.

It has been suggested (by, e.g., Low [2002] and others) that CMEs remove non-potential, helicity-carrying flux systems from the low corona and into the heliosphere, effectively ridding the erupting volume of magnetic helicity. This allows the magnetic field in the erupting region to relax to its global energetic minimum, the potential state. So, while reconnection *without* eruption can release some fraction of the corona's free energy, eruption (perhaps with attendant reconnection) can release all of the free energy.

Removing non-potential flux to infinity by ejection has an energetic cost, however, and the coronal field will not spontaneously erupt unless the energetic gain in relaxing to the potential state exceeds the energetic cost of ejecting the non-potential flux. Aly (1984,1991) and Sturrock (1991) argued that sufficient energy to open the field — a presumed requirement to eject a coronal flux system to infinity — cannot be stored in the field by twisting or shearing motions acting on the solar photosphere alone, assuming the field is force free and evolves ideally.

In a series of simulations, Linker, Amari, and their collaborators (Linker *et al.* 2001; Amari *et al.* 2003), have argued that cancellation simultaneously lowers $U_M^{(O)}$, the open field energy, while increasing U_F. In these numerical experiments, shear is applied to a model coronal arcade is sheared, followed by an imposed electric field on the bottom boundary, which leads to cancellation at the boundary and the formation of a flux rope. Given enough shear and cancellation, the flux rope erupts dynamically.

This paper is an analytic investigation of the effect of cancellation upon the open field energy and the free magnetic energy in sheared arcades. In Section 2., I define terms and discuss assumptions regarding cancellation. In Section 3., I show that cancellation tends to reduce the open field energy, and that cancellation in a sheared arcade can increase the coronal free energy. In Section 4., I show that the effective diffusion rate of horizontal field can differ substantially compared to the effective diffusion rate of the cancelling normal magnetic field, which has implications for the coronal field's evolution in response to cancellation. In Section 5., I use a simple model of a sheared arcade to illustrate these principles. Finally, in Section 6., I discuss the implications of this work, and address the issue of boundary conditions in simulations of cancellation.

1.1. Definitions of Cancellation

For this paper, which is not concerned directly with observations, I employ a more mathematical definition of cancellation: it occurs when oppositely signed magnetic fluxes threading a surface $\mathcal{S}$, in the presence of plasma, come into contact and equal amounts of oppositely signed flux "disappear" from $\mathcal{S}$ during a time interval Δt.

I assume that $\mathcal{S}$ separates two volumes, V and $\tilde{V}$, with the plasma β (the ratio of gas and magnetic pressures) low in the former and high in the latter. The magnetic Reynolds number is assumed large in both V and $\tilde{V}$, meaning the field evolution is nearly ideal. The normal field B_n is non-zero in at least one region of $\mathcal{S}$, and might or might not vanish on the rest of the bounding surface ∂V of V (which we often assume to be open). While I will use V and corona interchangeably, and $\mathcal{S}$ (the bottom boundary of V) and chromosphere interchangeably, the analysis presented here can be applied to flux cancellation in both other layers of the solar atmopshere and other physical contexts.

In reality, $\mathcal{S}$ is an atmospheric layer of finite thickness δz (almost certainly not constant in either space or time) over which the magnetogram field was derived from the emission and absorption of radiation. My approximation of $\mathcal{S}$ as a plane means that the magnetic field values I assume at $\mathcal{S}$ do not accurately represent the real field; I really refer to the spatially-averaged field that a magnetogram shows.

Probably the most straightforward criterion for determining whether cancellation is occuring is a global one: the total amount of unsigned flux through $\mathcal{S}$ decreases,

$$\frac{\partial|\Phi|}{\partial t} = \frac{\partial}{\partial t}\int d\mathcal{S}|B_n| < 0. \tag{1}$$

This integral constraint, however, is of limited utility in relating flux cancellation to the coronal energy budget.

Locally, we know that cancellation of flux in $\mathcal{S}$ must occur along a polarity inversion line (PIL), a zero contour of B_n. In the generic case (barring a special symmetry at the PIL), one can adopt a local coordinate system at a point $\mathbf{x}_0$ on the PIL, such that $\hat{x} \parallel \nabla B_n(\mathbf{x}_0)$, and $\hat{y} \cdot \nabla B_n(\mathbf{x}_0) = 0$. Essentially, $\hat{x}$ points from the negative side of the PIL toward the positive side, and $\hat{y}$ points to the right along the PIL to an observer standing on the positive side of the PIL, as illustrated in Figure 3. Generally, $\partial B_n/\partial y$ vanishes in this system. In what follows, this will be the default coordinate system.

Focusing on how magnetic flux is transported to the PIL, I define the *flux transport velocity*, which is equivalent to Démoulin and Berger's (2003) (Démoulin and Berger 2003) definition of the pattern velocity of magnetic flux at the photosphere,

$$\mathbf{u} \equiv \mathbf{v}_h - v_n \mathbf{B}_h / B_n. \tag{2}$$

(The subscript h on vector quantities refers to horizontal, by which I mean more precisely the component tangent to $\mathcal{S}$.) For Démoulin and Berger, $\mathbf{u}$ is the horizontal velocity at which magnetic features in a magnetogram of B_n *appear* to move when the evolution is ideal; $\mathbf{v}$ is the actual plasma velocity. Non-ideal terms also transport flux; their contribution is discussed further below. If the evolution is ideal, of tranport of magnetic flux into an area $d\mathcal{S}$ is given by

$$\partial \Phi_{d\mathcal{S}}/\partial t = - \int d(d\mathcal{S}) \nabla_h \cdot (\mathbf{u} B_n) = - \oint_{d\mathcal{S}} d\ell \, \hat{n} \cdot (\mathbf{u} B_n), \tag{3}$$

which applies whether the velocity $\mathbf{v}$ is purely vertical, purely horizontal, or has components in both directions. For example, this expression accurately captures the flux transport into $d\mathcal{S}$ from a moving, tilted magnetic flux tube, whether the tube is rising (or sinking), with $v_n \neq 0$ and $\mathbf{v}_h = 0$, or moving laterally, with $v_n = 0$ and $|\mathbf{v}_h| \neq 0$. This flux transport velocity describes the evolution of B_n in sequences of magnetograms that show cancellation whether reconnection, submergence, or emergence are at work. Since the evolution of B_n does not fully constrain $\mathbf{v}$, the actual plasma flow, I assume that $\mathbf{v}$ is known. (Recall that the plasma β in $\tilde{V}$, below $\mathcal{S}$, is assumed high, so the magnetic field within V exerts a negligible force on the plasma in $\tilde{V}$. Hence, the velocity at $\mathcal{S}$ is assumed to be kinematically imposed by flows in $\tilde{V}$, not dynamically determined by the state of the magnetic field in V.)

Fundamentally, the flux transport velocity perpendicular to the neutral line must be converging toward the PIL for cancellation to occur. In the local frame, $\partial u_x/\partial x < 0$ at the PIL is a necessary condition for cancellation to occur. In the archetypal convergence case, $u_x = 0$ at the PIL, which also makes it a velocity inversion line — a "VIL" for the flux transport velocity perpendicular to the PIL. In the general case, the flux at $\mathbf{x}_0$ is being transported with a velocity $\mathbf{u}_0$; a Galilean transformation to the the comoving frame restores $u_x = 0$ at the PIL.

A useful local criterion for flux cancellation is that

$$B_n u_x < 0 \tag{4}$$

is satisfied on both sides of the PIL in the comoving frame. Since only flux tranport perpendicular to the PIL instantaneously leads to flux cancellation, flux transport in the $\hat{y}$ direction (parallel to the PIL) does not contribute to cancellation (to first order in Δt), though it can affect the magnetic energy.

How does the plasma velocity $\mathbf{v}$ relate to the flux transport velocity $\mathbf{u}$ when cancellation is occurring? If the normal velocity, v_n, vanishes at the PIL, v_x carries normal flux toward the PIL, where it reconnects, as described elsewhere (e.g., Priest and Forbes, 2002). If the evolution is ideal, then the vertical velocity must be non-zero at the PIL, and the disappearance of vertical flux arises by submergence of inverted-U-shaped field lines (Chae *et al.* 2004; Harvey *et al.* 1999; Rabin *et al.* 1984) or emergence of U-shaped field lines (van Driel-Gesztelyi *et al.* 2000). Essentially, vertical flux smoothly changes to horizontal flux, which is then advected away from $\mathcal{S}$.

2. Cancellation & Field Energy

2.1. Open Field Energy

The open field state is one in which the field in the open subvolume of the corona is current-free, except at separatrix surfaces between oppositely-directed flux systems. (Here, open is not used in its usual mathematical sense; rather, the adjective open describes flux systems in which field lines extend to infinity.)

Assuming the field in the entire solar corona is open, the field can easily be computed from the normal flux distribution B_n on $\mathcal{S}$: B_n is replaced with $\tilde{B}_n = |B_n|$; the open scalar potential $\chi^{(O)}$ is then derived from $\tilde{B}_n$; finally, the directions of field lines originating in regions where $B_n < 0$ are reversed. The energy present in such an open configuration can then be easily calculated in terms of a surface integral of the scalar potential of the open field, $\chi^{(O)}$, on the boundary,

$$U_M^{(O)} = (1/8\pi) \int dV\, \mathbf{B}^{(O)} \cdot \mathbf{B}^{(O)} = (1/8\pi) \int dV\, (\nabla\chi^{(O)} \cdot \nabla\chi^{(O)})$$

$$= (1/8\pi)\int d(\partial V)\,\chi^{(O)}\,(\hat{n}\cdot\nabla\chi^{(O)}) - (1/8\pi)\int dV\,(\chi^{(O)}\nabla^2\chi^{(O)})$$
$$= (1/8\pi)\int d(\partial V)\,\chi^{(O)}\,(\partial\chi^{(O)}/\partial n) = \frac{1}{8\pi}\frac{\partial}{\partial n}\int d(\partial V)\frac{(\chi^{(O)})^2}{2}. \qquad (5)$$

In a spherical harmonic expansion of $\chi^{(O)}$,

$$\chi^{(0)}(r,\theta,\phi) = R_\odot \sum_{lm} \frac{R_\odot^{(\ell+1)}}{r^{(\ell+1)}(\ell+1)} \left(\int d\Omega' \tilde{B}_n(\theta',\phi') Y^*_{\ell m}(\theta',\phi')\right) Y_{\ell m}(\theta,\phi) \qquad (6)$$

the leading order term is proportional to the integrated absolute flux (the monopole moment). The integral definition of cancellation in equation (1) ensures that this term diminishes as flux cancels, so the leading-order change in energy is

$$\Delta E_0^{(0)} = 2(\Delta\Phi)^2/R_\odot, \qquad (7)$$

where $\Delta\Phi$ is the amount of flux that cancelled in *one* polarity. While convergence of oppositely signed fluxes, a necessary precursor to cancellation, lowers the potential energy (by analogy with electrostatics), it raises the open field energy, causing higher order terms in the expansion above to be enhanced (though their amplitude decreases as $[\ell+1]^{-1}$). When the converging fluxes have cancelled completely, however, the enhancement to the higher order terms disappears. Nonetheless, I can only say that cancellation *tends* to lower the open field energy. Observationally, Chae *et al.* (2001) report a loss of 1.5×10^{21} Mx over three days due to cancellation in AR 8668, which corresponds to a leading-order change in the open field of $\sim 6\times10^{31}$ erg, on the order of energy changes associated with flares and CMEs.

The energy of an entirely open corona is, however, usually irrelevant; rather, the energy present when a particular subdomain of the corona is open interests us more. The relative change in the open field energies before and after cancellation, $\Delta U^{(O)}/U_M^{(O)}$, can be larger when considering a subvolume of the corona. Calculating this energy, however, requires solving a boundary value problem where the boundaries (the separatrices between the volume that would open and the regions that would remain closed) are free surfaces, and is, in general, analytically intractable. Nonetheless, the leading-order change in open field energy from cancellation will still be proportional to the square of the cancelled flux.

2.2. Poynting Flux

Rather than calculate the field evolution in V in response to boundary evolution that consistent with one of the cancellation scenarios above, I will calculate the Poynting flux on the boundary, to determine, where possible, changes in the field energy. Assuming the electric field $\mathbf{E}$ can be written

$$\mathbf{E} = -(\mathbf{v} \times \mathbf{B})/c + \eta(\nabla \times \mathbf{B})/c, \tag{8}$$

where $\eta = c^2/4\pi\sigma$ is constant, then the normal Poynting flux S_n of energy into V across $\mathcal{S}$ can be written

$$S_n = \frac{c}{4\pi}\hat{n} \cdot (-(\mathbf{v} \times \mathbf{B})/c + \eta(\nabla \times \mathbf{B})/c) \times \mathbf{B}, \tag{9}$$

where I have only kept the resistive term in Ohm's Law. If the field is force-free, or the diffusivity is small enough, then the η term can be ignored. Given that some cancellation is reconnective, however, I keep it, but express it interms of $\mathbf{J}$,

$$4\pi S_n = v_n(\mathbf{B}_h \cdot \mathbf{B}_h) - B_n(\mathbf{v}_h \cdot \mathbf{B}_h) + (c/\sigma)(\mathbf{J}_h \times \mathbf{B}_h). \tag{10}$$

For $S_n < 0$, the magnetic energy in V decreases.

Since, as discussed above, only flux tranport perpendicular to the PIL leads to flux cancellation, flows in the $\hat{y}$ direction (parallel to the PIL) do not instantaneously contribute to cancellation, though they may contribute to the total Poynting flux.

As an aside, I note that B_n is zero *at* the PIL, so the magnetic field near the PIL must be expanded in a Taylor series in the coordinate perpendicular to the PIL,

$$B_n \sim \delta\mathbf{x} \cdot \nabla B_n, \tag{11}$$

to get the Poynting flux density near the PIL. The areal integral over S_n in the cancelling region will, in general, be nonzero.

2.3. Potential Field Energy

The change in magnetic energy of the potential field cannot be calculated using the Poynting flux, since physical equations (e.g., the induction equation) do not govern the evolution between the initial and final potential fields, $\mathbf{B}_i^{(P)}$ and $\mathbf{B}_f^{(P)}$, in V: the former matches one boundary condition, the latter matches another, but the fields in V do not necessarily evolve continuously. It is still the case, however, that the induction equation, using the actual tangential magnetic field and the actual plasma flow, governs the field's evolution on the boundary. Fortunately, this is all we need to recover the leading-order change in energy in the potential field.

One can express $\mathbf{B}_f^{(P)}$ as $\mathbf{B}_i^{(P)}$ plus a change in the field, $\delta\mathbf{B}$, where

$$\nabla \times \delta\mathbf{B} = 0 \Longrightarrow \delta\mathbf{B} = -\nabla\chi_\delta \tag{12}$$

$$\nabla \cdot \delta\mathbf{B} = 0 \Longrightarrow \nabla^2\chi_\delta = 0 \ , \tag{13}$$

since $\mathbf{B}_i^{(P)}$ and $\mathbf{B}_f^{(P)}$ are both divergence-free and potential. For small changes in the field δB, the change in in potential field energy is then, to leading order,

$$\delta U_m^{(P)} = \frac{1}{8\pi}\int dV\,(\mathbf{B}_f^{(P)})^2 - (\mathbf{B}_i^{(P)})^2 \tag{14}$$

$$\simeq \frac{1}{4\pi}\int dV\,\mathbf{B}_i^{(P)}\cdot\delta\mathbf{B} = \frac{1}{4\pi}\int dV\,(\nabla\chi_i\cdot\nabla\chi_\delta) \tag{15}$$

$$= \frac{1}{4\pi}\int d(\partial V)\,\chi_i\,\frac{\partial\chi_\delta}{\partial n} - \frac{1}{4\pi}\int dV\,(\chi_i\nabla^2\chi_\delta)$$

$$= \frac{1}{4\pi}\int d\mathcal{S}\,\chi_i\,\frac{\partial\chi_\delta}{\partial n} \ . \tag{16}$$

In the final step, I kept only the surface term corresponding to the changes in magnetic field on $\mathcal{S}$, on the assumption that the changes in the magnetic field on the other components of the bounding surface, ∂V, are negligible. Recognizing that $\partial\chi_\delta/\partial n = \delta B_n$ (this derivative is outward normal), with a time step δt, one can refer to the finite difference approximation to the induction equation (with η constant, and the actual $\mathbf{B}_h$),

$$\frac{\delta B_n}{\delta t} = \nabla_h\cdot(v_n\mathbf{B}_h - \mathbf{v}_h B_n) - (c/\sigma)(\nabla_h\times\mathbf{J}_h) \ , \tag{17}$$

to write

$$\delta U_m^{(P)} = \frac{\delta t}{4\pi}\int d\mathcal{S}\,\chi_i\,\nabla_h\cdot(v_n\mathbf{B}_h - \mathbf{v}_h B_n) - \frac{c\delta t}{4\pi\sigma}\int d\mathcal{S}\,\chi_i\,(\nabla_h\times\mathbf{J}_h) \tag{18}$$

$$\frac{\delta U_m^{(P)}}{\delta t} = -\frac{1}{4\pi}\int d\mathcal{S}\,\nabla_h\chi_i\cdot(v_n\mathbf{B}_h - \mathbf{v}_h B_n) + \frac{c}{4\pi\sigma}\int d\mathcal{S}\,(\nabla_h\chi_i\times\mathbf{J}_h)$$

$$+ \frac{1}{4\pi}\oint_{\mathcal{S}} d\ell\,\chi_i\hat{n}_{\mathcal{S}}\cdot(v_n\mathbf{B}_h - \mathbf{v}_h B_n) - \frac{c}{4\pi\sigma}\oint_{\mathcal{S}} d\hat{\ell}\cdot(\chi_i\mathbf{J}_h) \tag{19}$$

$$\frac{\delta U_m^{(P)}}{\delta t} = \frac{1}{4\pi}\int d\mathcal{S}\,\mathbf{B}_{h,i}^{(P)}\cdot(v_n\mathbf{B}_h - \mathbf{v}_h B_n) - \frac{c}{4\pi\sigma}\int d\mathcal{S}\,(\mathbf{B}_{h,i}^{(P)}\times\mathbf{J}_h) \ , \tag{20}$$

where I have assumed that the field is sufficiently localized on $\mathcal{S}$ that we can neglect the line integrals in the partial integrations on the boundaries of $\mathcal{S}$. I can now express the leading order change in the potential field's magnetic energy as a Poynting-like energy flux density that depends upon both the actual and potential horizontal fields,

$$4\pi S_n^{(P)} = \mathbf{B}_{h,i}^{(P)}\cdot(v_n\mathbf{B}_h - \mathbf{v}_h B_n) + (c/\sigma)(\mathbf{J}\times\mathbf{B}_{h,i}^{(P)}) \ . \tag{21}$$

I note that any additional term(s) from the generalized Ohm's Law kept in equation (10) would, via equation (17), appear in equation (21) with $\mathbf{B}_h \to \mathbf{B}_{h,i}^{(P)}$.

2.4. Free Energy

To determine the rate of change of free magnetic energy in the corona, I now examine the difference between S_n and $S_n^{(P)}$,

$$4\pi(S_n - S_n^{(P)}) = (v_n B_x - B_n v_x)(B_x - B_x^{(P)}) + v_n B_y (B_y - B_y^{(P)}) - cJ_y(B_x - B_x^{(P)})/\sigma + cJ_x(B_y - B_y^{(P)})/\sigma \ , \quad (22)$$

one term at a time. To make progress, I make the following assumptions about $\mathbf{B}$ and $\mathbf{B}^{(P)}$,

$$B_x^{(P)} < 0 \quad (23)$$

$$|B_y^{(P)}| \ll |B_y| \ , \quad (24)$$

which are appropriate for a sheared arcade.

I assume the criterion for flux cancellation, equation 4), is satisfied, so $-B_n u_x = (v_n B_x - v_x B_n) > 0$ near the neutral line.

I next consider the sign of difference between the actual and potential horizontal fields perpendicular to the PIL, $(B_x - B_x^{(P)})$. If field lines arch over the PIL, from $B_n > 0$ to $B_n < 0$ across the PIL, then B_x points in the conventional sense, and $B_x < 0$ in the local frame. If, however, B_x points in the "inverse" direction, such that $B_x > 0$ at the PIL, then field lines are dipped over the PIL. Dipped field lines over the PIL occur, e.g., at a bald patch in a potential field (Titov *et al.* 1993), or when the lowest field line in a horizontal flux rope osculates $\mathcal{S}$ in a current-carrying configuration. Figure 2 shows two simple configurations in which the sign of B_x varies, based upon the field's topology above the PIL. Ignoring potential field configurations with dipped field lines over the PIL, I assume that field lines in $\mathbf{B}^{(P)}$ generally arch over the PIL conventionally, consistent with equation (23).

If the actual field lines dip over the PIL, $B_x > 0$, as with a flux rope, and field lines of $B_x^{(P)}$ arch over the PIL, then the difference $(B_x - B_x^{(P)})$ in equation (22) is positive.

In a typical bipolar arcade, where both the actual and potential field lines arch over the PIL in the conventional sense, both $B_x < 0$ and $B_x^{(p)} < 0$. In a sheared bipolar arcade, the actual field has a component along the PIL, $B_y \neq 0$, while the potential field is essentially perpendicular to the PIL, $B_y \simeq 0$. Klimchuk (1990) analyzed such configurations and found that increasing shear — essentially, increasing $|B_y|$ — invariably increases the height at which field lines of $\mathbf{B}$ cross the PIL, a result consistent with other models of such configurations. Since the potential state has minimal shear, this expansion of the arcade has the consequence that the average flux density perpendicular to the PIL, B_x, decreases compared to the potential state, so

$$|B_x^{(P)}| > |B_x| \tag{25}$$

usually obtains at the PIL.

As an aside, I note that the direction of B_x at $\mathcal{S}$ is poorly defined when the field near the PIL exhibits an X-point topology, like the configurations shown in Figure 1, since $B_x \sim 0$ near the PIL in such cases. The preceding discussion suggests that this does not matter when calculating the energy flux, since the potential field does have a well defined direction, by assumption.

In any case, then, the $(B_x - B_x^{(P)})$ term in equation (22) is positive, whether the horizontal field perpendicular to the PIL points in the conventional or inverse sense, and $(B_x - B_x^{(P)})(v_n B_x - v_x B_n) > 0$ for cancellation in both cases.

Equation (24) implies the second term in equation (22), $v_n B_y(B_y - B_y^{(P)})$, has the same sign as v_n.

Finally, I consider the two non-ideal terms in equation (22). Since J_x and J_y involve vertical derivatives of the magnetic field's horizontal components, these two quantities are not currently available with most vector magnetogram observations, which invert Stokes' profiles over a single, relatively thin, atmospheric layer (but see Metcalf [1995]). Nonetheless, I can make quantitative staments about the sign of these terms, given various assumptions.

Since $\partial B_z/\partial x > 0$ by assumption in the local coordinate system, I can assume $J_y \propto (\partial B_x/\partial z - \partial B_z/\partial x) \leq 0$ obtains for typical sheared arcades. In the potential field, $\partial B_x^{(P)}/\partial z > 0$ must be true to satisfy $J_y^{(P)} = 0$. In a sheared field, as discussed above, the shear-driven expansion causes B_x across the PIL to decrease as the arcade expands, meaning $\partial B_x/\partial z < \partial B_x^{(P)}/\partial z$. This last relation is also true when a flux rope lies over the PIL. These arguments, combined with arguments presented above, suggest $-J_y(B_x - B_x(P)) > 0$ is usually true.

Since $\partial B_z/\partial y = 0$ by assumption in the local coordinate system, $J_x \propto -\partial B_y/\partial z$ must be true. I assume that, as above, $B_y^{(P)} \simeq 0$, leaving $J_x(B_y - B_y^{(P)}) \propto -\partial B_y^2/\partial z$. Martin (1998) (Martin 1998) has observed that cancellation usually occurs between plage fields. Lites (2005), in observations of photospheric vector magnetograms, found plage fields far from PILs are nearly vertical (implying $\partial|\mathbf{B}_h|/\partial z > 0$ must obtain there), but that plage fields near PILs are typically horizontal. Hence, observations do not directly constrain $\partial B_y^2/\partial z$ at the photosphere in cancelling regions. Observations of prominence fields overlying PILs, however, show that $\partial|\mathbf{B}_h|/\partial z < 0$ (Rust 1967; Kim *et al.* 1984). If this property of the coronal fields above PILs also applies at photosphere, then $J_x(B_y - B_y^{(P)}) < 0$.

What is the total contribution from these two, oppositely signed non-ideal terms? While it is far from clear that real solar fields are force-free at the photosphere (Metcalf *et al.* 1995; Moon *et al.* 2002), progress can be made if I assume they *tend* to be, since I am only interested in the relative signs of the current and field components. Proceeding in this vein reduces both non-ideal terms in equation (22) to

$$-J_y(B_x - B_x^{(P)}) + J_x(B_y - B_y^{(P)}) = J_y B_x^{(P)}) - J_x B_y^{(P)}) \simeq J_y B_x^{(P)} > 0 \ , \qquad (26)$$

where, as above, $B_y^{(P)} \simeq 0$, $J_y < 0$, and $B_x^{(P)} < 0$ were assumed.

In the following sections, I examine the import of the results derived in this section when more specific assumptions are made about the velocity and magnetic fields.

3. Flux Transport Rates

An essential difference between ideal cancellation and reconnective cancellation is the transport of flux, Φ_y, that threads the $x - z$ plane (or runs parallel to the PIL) in the local frame. With either ideal submergence of inverse-U field lines or ideal emergence of U-shaped field lines, Φ_y is merely advected with the flow. With reconnective cancellation, however, both diffusion and material flow transport Φ_y away from $\mathcal{S}$. Hence, while the transport of horizontal flux seems straightforward in the ideal cases, horizontal flux transport in the reconnective case is not so clear. How does the rate of diffusion of Φ_y compare with the rate of cancellation of normal flux, Φ_n? Does Φ_y diffuse asymmetrically in the direction normal to $\mathcal{S}$? How does B_y evolve above and below $\mathcal{S}$ as the reconnection proceeds?

Assuming constant diffusivity, one can estimate the diffusion rate of Φ_y using knowledge of the cancellation rate of Φ_n. I assume steady-state reconnective cancellation from a purely horizontal flow at $\mathcal{S}$, with a vertically-oriented current sheet of thickness $2\delta x$, vertical extent $2\Delta z_P$ (on the order of the local pressure scale height), and no variation in $\hat{y}$, after Litvinenko (1999). The rate of flux transport from the ideal region is balanced by the diffusive loss terms,

$$0 = \frac{\partial \Phi_n}{\partial t} L_y^{-1} \qquad (27)$$

$$= B_n v_x + \eta \frac{\partial B_n}{\partial x} + \eta \int dx \frac{\partial^2 B_n}{\partial n^2} \ . \qquad (28)$$

Given the X-type field topology perpendicular to $\hat{y}$, the $\partial^2 B_n/\partial n^2$ curvature term essentially means that field lines slip through the plasma, away from the PIL, with an effective diffusive velocity $v_x^{(D)}$ that opposes the advective inflow, v_x. Consequently, neglecting this term results in an overestimate of the flux transported into the diffusion region, resulting in an overestimate of the effective diffusivity. I want to put an upper bound on the diffusion rate of Φ_y compared to Φ_n, so an error in this sense is acceptable. Approximating $\partial B_n/\partial x \sim B_n/\delta$ implies

$$\eta \lesssim |v_x|\delta_x \ , \tag{29}$$

which can be used to estimate the diffusion rate of Φ_y,

$$\frac{\partial \Phi_y^{(D)}}{\partial t} \simeq \eta \left(\frac{\partial B_y}{\partial x} \Delta z_P + \frac{\partial B_y}{\partial z} \delta x \right) \tag{30}$$

$$\lesssim v_x \delta x \left(\frac{\partial B_y}{\partial x} \Delta z_P + \frac{\partial B_y}{\partial z} \delta x \right) \ . \tag{31}$$

Hence, the diffusive transport of Φ_y is governed by gradients in B_y across the diffusion region, about which very little can be said in general. Observations often show B_y takes on extremal values near the PIL, with no indication of reversal in B_y across the PIL; hence, the $(\partial B_y/\partial x)\delta x$ term can probably be neglected.

The $(\partial B_y/\partial z)\Delta z_p$ term is trickier. In many numerical simulations of magnetic fields in the outer solar atmosphere, all components of the initial magnetic field decay monotonically with increasing height (e.g., in potential and many constant-α force free fields), implying $\partial |B_y|/\partial z < 0$. On the Sun, $|\mathbf{B}_h| \sim 0$ below the chormospheric merging layer over large regions of the surface, while $|\mathbf{B}_h| \neq 0$ in the atmosphere above this layer, implying $\partial |B_y|/\partial z > 0$ in these regions. Observations by Lites (2005), however, show that plage fields near PILs are often horizontal, meaning this fact does not apply to cancelling fields. As above, extrapolating coronal magnetic field observations showing $\partial |\mathbf{B}_h|/\partial z > 0$ down to the photosphere implies Φ_y could diffuse downward in the presence of resistivity.

I note that assuming Δz_P is less than a pressure scale height implies that, while a vertical pressure gradient exists at $\mathcal{S}$, this gradient does not affect the flux transport. Clearly, more work remains for both observers and theoreticians.

Though Φ_y is conserved during the reconnection, B_y is not: parallel flux advected through the diffusion region is free to rotate to a different orientation in the relaxation process. Here, the evolution of B_y is quite different above and below the reconnecting region, since reconnected field lines above and below the X-point are anchored in very different locations, from the nature of the sheared arcade in which the fields are reconnecting: footpoints of field lines below the cancellation site are closer together in y than the foot points of field lines above the X-point. Hence, the lower field lines make larger angles with the PIL than the higher field lines do. Hence, $|B_y|$ can increase above the reconnection site faster than B_y below it does, even if more flux Φ_y diffuses downward than upward.

4. Hypothetical Evolution in Simple Arcade Model

The physical picture in this model is that, in response to the normal field evolution due to the cancellation, $B_0 \rightarrow B_0'$, the coronal field relaxes between cancellation events, at which point another cancellation begins and the process is repeated.

The coronal field can either evolve ideally or via fast magnetic reconnection, both of which preserve the magnetic helicity. The Taylor-Woltjer theorem implies that the coronal field will relax to another constant-α state, with a different force-free parameter ($\alpha \longrightarrow \alpha'$), but with the same helicity as the pre-cancellation field.

We can calculate the vector potential,

$$A_y = B_0 \sin(kx) e^{-\ell z}/k \tag{32}$$

$$A_z = \alpha B_0 \sin(kx) e^{-\ell z}/k^2 \ , \tag{33}$$

so we can calculate the gauge-invariant magnetic helicity,

$$\mathcal{H} = \frac{\pi \alpha B_0^2 L}{k_z k_x^3} \ . \tag{34}$$

Cancellation in this model leads to a decrease in B_0, hence an increase in α for constant helicity. (Actually, a [non-linear force-free] Gold-Hoyle flux rope would be better as a model than this example, but I'm still working on that.)

Hypothesis: the presence of a component of the field parallel to the PIL causes the PIL to evolve differently in the actual and potential field cases, even if the velocity field causing the evolution is identical.

5. Discussion

For $v_n = 0$ and v_x converging toward the PIL, as when a flux rope has been formed by steady reconnective cancellation in a sheared arcade, (Linker *et al.* 2001; Amari *et al.* 2003), the ideal terms increase the coronal free energy.

For cancellation with $v_n > 0$, the ideal terms also increase the coronal free energy, just as one would expect for, e.g., an emerging flux rope (Rust and Kumar 1994). For $v_n < 0$, and $v_x = 0$, the ideal terms decrease the coronal free energy, as one would expect for a submerging flux rope. For $v_x \neq 0$ and $v_n < 0$, the change in coronal free energy from the ideal terms is not certain.

For reconnective cancellation, with $v_x \neq 0$ and $v_n = 0$, the ideal contributions increases the coronal free energy, since the only remaining term in equation (22), $-v_x B_n$, is also positive, by assumption.

For cancellation by submergence and emergence, with $v_n \neq 0$ and $v_x = 0$,

$|B_y|$ — tends to cause the height at which field lines cross the PIL to increase, thereby lowering the average flux density across the PIL at a given height, and at $\mathcal{S}$ in particular. This analysis assumes the potential field has less shear than the actual field

If equation (25) is correct, then the left hand side of equation (40) is *positive*, and cancellation appears to increase the coronal free energy, regardless of whether flux is cancelling via reconnection or submergence. Such a conclusion would be troubling, because removing non-potential fields from the corona by submergence certainly lowers the free magnetic energy there!

By examining the reconnection and submergence cases separately, one can resolve this seeming paradox. I illustrate this points with a well known model of a constant-α, sheared arcade in Cartesian geometry,

$$B_x = -(k_z/k_x) B_0 \sin(k_x x) \exp -k_z z \quad (35)$$

$$B_y = -(\alpha/k_x) B_0 \sin(kx) \exp -k_z z \quad (36)$$

$$B_z = -B_0 \cos(kx) \exp -k_z z, \quad (37)$$

where the force-free parameter is $\alpha \equiv \sqrt{k_x^2 - k_z^2}$, and the field is invariant in y and periodic in x. In this simple model, field lines arch over the PIL, at $k_x x = \pi/2$, and lie in vertical planes oriented at an angle $\tan^{-1}(B_y/B_x)$ with respect to the $x - z$ plane.

The field component along the PIL is determined by α. The potential case corresponds to $\alpha = 0 = B_y$, so $k_z = k_x$ obtains, and the field lines are perpendicular to the PIL. As $|\alpha|$ increases and B_n kept fixed at the photosphere, k_z decreases (k_x is fixed), and field lines in the arcade rise. Decreasing k_z means B_x at the $x = 0$ plane decreases, too, because the height at which any given field line crosses $x = 0$ increases with increasing α, thereby diminishing the average flux density normal to the $x = 0$ plane.

Now consider cancellation by submergence, with $v_x = 0$, and $v_n = -v_0 < 0$. First, more flux parallel to the PIL is being submerged in the non-potential case, though it does not contribute to cancellation in this (artificial) 2.5D example. Rewriting equation (40), at $x = 0 = z$, in terms of the field components gives

$$4\pi(S_n - S_n^{(P)}) = -v_0(B_x^2 + B_y^2 - B_x^{(P)}) \quad (38)$$

$$= -v_0((k_z^2 + \alpha^2)B_0^2/k_x^2 - k_z B_0^2/k_x) \quad (39)$$

$$= -v_0 B_0^2(1 - k_z/k_x) < 0 \ , \quad (40)$$

So the free energy is dropping. Further, the weaker B_x in the actual field at the PIL means submerging a given amount of the horizontal flux $\delta\Phi_x$ (equivalent to cancelling a given amount of normal flux $\delta\Phi_n$), requires more time (or a stronger downflow) in the actual case than in the potential case.

Next, consider cancellation by reconnection at the PIL, with $v_n = 0$, and v_x converging toward the PIL.

6. Discussion

Not surprisingly, cancellation via emergence and submergence of non-potential fields tends to add and remove, resp., free energy from the corona. What's less clear is that reconnective cancellation does so, the simulations of Linker *et al.* and Amari *et al.* notwithstanding.

Reasons I think prominence fields are formed by reconnection in the corona, vs. emergence of fully-formed flux ropes: 1) a majority of prominences are formed on the boundaries between active regions (Tang 1987); 2) the hemishperic preference rules for active regions are much weaker than for prominences, as noted by Pevtsov (private communication).

Fig. 1 — Zwaan's (1987) three scenarios to explain cancellation in terms of reconnection, which vary in the altitude difference between the magnetograph's imaging layer (thick solid line) and the reconnection site (the X-point formed by the thin, dashed lines). The thick, white arrows illustrate the velocity of the reconnecting flux.
Fig. 2 — a) The normal magnetic field B_n near the PIL in a plane parallel to and two units away from a line current. b) Selected field lines if the current source is below the plane. Cancellation of normal flux in the plane would decrease the field energy above the bottom boundary in this case. c) Selected field lines arising from the same

boundary condition, but with the current source above the plane. Cancellation of normal flux in the plane would increase field energy above the bottom boundary.

Fig. 3 — Grayscale indicates B_n in this schematic representation of the local coordinate system at a point $\mathbf{x}_0$ on the PIL (dashed line). Essentially, $\hat{x} \parallel \nabla B_n$ points from the PIL's negative side toward the positive side, and $\hat{y}$ points to the right along the PIL to an observer standing on the positive side of the PIL.

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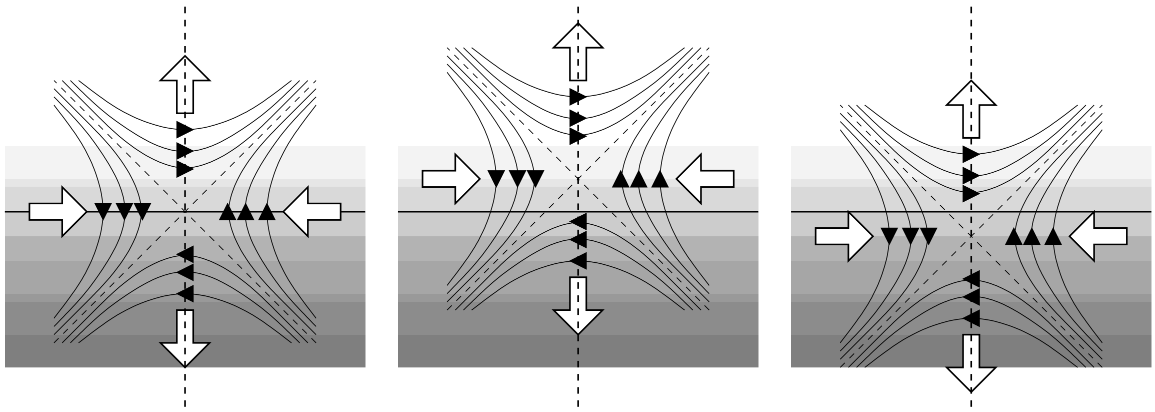

Fig. 1.—

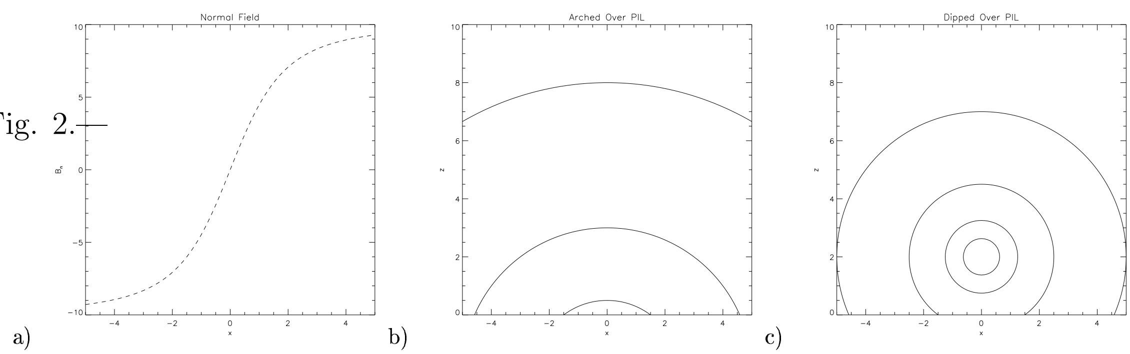

Fig. 2.—

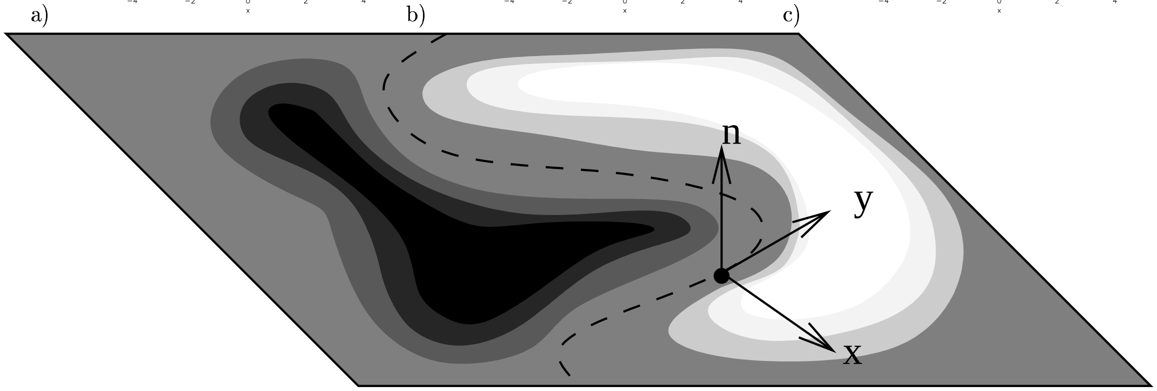

Fig. 3.—

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