The Determination and Use of Mean Electron Flux Spectra in Solar Flares

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ABSTRACT

Hard X-ray spectra in solar flares provide information on electron acceleration and propagation processes. We here point out that the inference of these processes involves two distinct steps: first, the model-*independent* deconvolution of the hard X-ray spectrum to obtain the effective mean electron spectrum $\bar{F}(E)$ in the source; and second, the model-*dependent* interpretation of this mean spectrum in terms of physical processes operating in that source. Thus, the mean electron spectrum is a natural "middle ground" on which to compare the predictions of models with observations, and we urge the presentation of results, both from analysis of photon spectra and from modeling of candidate physical processes, in the form of $\bar{F}(E)$ spectra. We consider the constraints that various source models impose on $\bar{F}(E)$ and we present explicit forms for an illustrative $\bar{F}(E)$ corresponding to the injection of a power-law spectrum of electrons into a thick target of nonuniform ionization level.

Subject headings: Sun: flares

1. Introduction

Much attention has been paid to the inference of accelerated electron spectra and/or temperature distributions from high-resolution hard X-ray spectra such as from RHESSI (e.g., Piana & Brown 1998). Such analyses involve a variety of assumptions regarding (a) the emission process, and in particular the bremsstrahlung cross-section used, and (b) modelling factors such as the location of the electron acceleration region and physical conditions in the propagation region. Due to the different extent to which such factors have been considered, it is sometimes challenging to compare results.

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To improve upon this unnecessarily confusing situation, in this *Letter* we point out that the basic information contained in *any* observed hard X-ray spectrum $I(\epsilon)$ is the *mean electron flux spectrum* $\bar{F}(E)$ in the source. Determination of $\bar{F}(E)$ from observed hard X-ray spectra depends only on the bremsstrahlung cross-section used and involves no assumptions whatsoever about the acceleration or propagation of the electrons in the target. Conversely, theoretical prediction of $\bar{F}(E)$ involves no assumptions about the cross-section and depends only on physical processes in the source. The quantity $\bar{F}(E)$ therefore represents the natural "middle ground" for comparison of models with observation, and hence we urge its adoption as a fiducial reference for both observational analyses of hard X-ray spectra (see Holman et al. 2003; Piana et al. 2003) and theoretical models of electron dynamics (see Emslie 2003).

2. Inference of Mean Electron Flux Spectra from Hard X-ray Spectra

Following Brown (1971), we define the mean target proton density $\bar{n} = V^{-1} \int n(\mathbf{r})\, dV$ and write the photon spectrum at distance R as

$$I(\epsilon) = \frac{1}{4\pi R^2}\, \bar{n} V \int_{\epsilon}^{\infty} \bar{F}(E)\, Q(\epsilon, E)\, dE, \tag{1}$$

where the cross-section $Q(\epsilon, E)$ includes the effect of heavy ions in the target and $\bar{F}(E)$ is the *mean electron flux spectrum* defined by

$$\bar{F}(E) = \frac{1}{\bar{n} V} \int_V F(E, \mathbf{r})\, n(\mathbf{r})\, dV. \tag{2}$$

(Equation [1] assumes that the cross-section is a function only of E and ϵ and so is assumed isotropic and unpolarized.) Equation (1) is a Volterra integral equation for the mean electron flux spectrum $\bar{F}(E)$ corresponding to an observed photon flux spectrum $I(\epsilon)$. Its solution is problematic (e.g., Craig & Brown 1976), requiring algorithms to suppress noise amplification while preserving information on the source function $\bar{F}(E)$ (e.g., Piana & Brown 1998). However, the crucial point is that *the determination of $\bar{F}(E)$ depends only on the form of $Q(\epsilon, E)$; it assumes nothing about the physical processes operating in the source.*

What is the significance of $\bar{F}(E)$? It is the electron spectrum that would be required to produce the observed photon spectrum in a homogeneous source of proton density $\bar{n}$ and volume V. Equivalently, it is the mean electron flux spectrum incident upon a target proton in an inhomogeneous source. Johns & Lin (1992) refer to this quantity as the "X-ray-producing electron spectrum." From a theoretical viewpoint, this spectrum is determined by many physical properties, such as the location of, and spectrum from, the acceleration region and the form of subsequent energy losses. Observationally, however, it is the sole quantity that can be inferred from a photon spectrum without knowledge of any of these factors: once $\bar{F}(E)$ has been found from inversion of Equation (1), all subsequent interpretation of $\bar{F}(E)$ concerns only physical processes in the source and not the form of $Q(\epsilon, E)$ chosen. We therefore recommend that high quality hard X-ray spectra, such as from RHESSI, be inverted (or forward-fitted) to obtain $\bar{F}(E)$ (or, with a simple scaling, the target-integrated spectrum $\int F(E, \mathbf{r})\, n(\mathbf{r})\, dV$), and that further (source-model-dependent) analyses be separated from this initial step. Likewise, we recommend that modelers aim at synthesizing $\bar{F}(E)$ (or $\int F(E, \mathbf{r})\, n(\mathbf{r})\, dV$) forms to be compared with observationally-inferred ones. (To ease intercomparison of results, we also suggest that observational inference of $\bar{F}(E)$ should utilize a standardized [isotropic] cross section $Q[\epsilon, E]$.) Through a such a standardization, we can greatly facilitate intercomparison of results and also maintain an appropriate degree of independence between observational and theoretical analyses.

Solar hard X-rays also include a photospheric albedo contribution. Ideally, one should deconvolve the primary source spectrum from the total observed spectrum before proceeding with (1). However, this process is inherently nonlinear and difficult to carry out. We therefore propose that $\bar{F}(E)$ be calculated by a straightforward inversion of the *total* observed photon spectrum $I(\epsilon)$ and that interpretation of the resulting form of $\bar{F}(E)$ (the *effective* mean electron flux spectrum) take into account the effect of albedo (Alexander & Brown 2002).

3. An Example of a Mean Electron Flux Spectrum and its Interpretation

Determination of $\bar{F}(E)$ from Equation (1) is mathematically equivalent to determining the electron flux spectrum corresponding to a thin-target $I(\epsilon)$ – a well-understood problem (e.g., Brown 1971). Forms of $\bar{F}(E)$ inferred from hard X-ray data appear in companion papers (Holman et al. 2003; Piana et al. 2003). We here illustrate the theoretical construction of an $\bar{F}(E)$ by considering the case of a beam of electrons traversing a target with a spatially nonuniform ionization.

Electrons traversing a neutral target suffer energy losses reduced by a constant factor ($\simeq 3$) relative to an ionized target (e.g., Brown 1973; Emslie 1978). Electrons therefore have a higher bremsstrahlung "efficiency" in partially-ionized regions. Following Brown et al. (1998), Kontar, Brown, & McArthur (2002; hereafter KBM) calculated the bremsstrahlung emission resulting from a power-law injection spectrum ($F_o[E_o] = CE_o^{-\delta}$) for a target consisting of an ionized corona situated above a neutral chromosphere, neglecting scattering effects. If we write $n\, dV = A\, dN$, where A is the flare area (cm^2) and N the column density (cm^{-2}) normal to that area, comparison of Equation (1) with Equation (7) of KBM shows that

$$\bar{F}(E) \sim A \int_0^\infty F(E,N)\,dN = \frac{A\,C\,E^{2-\delta}}{(\delta-1)K}\left[1+\nu\left(1+\frac{E_*^2}{E^2}\right)^{\frac{1-\delta}{2}}\right]. \quad (3)$$

Here $E_*^2 = 2KN_*$, where N_* is the column density at which the ionization level changes abruptly and $K = 2\pi e^4 \Lambda$ (with e the electronic charge and Λ the Coulomb logarithm for an ionized target). Kontar et al. (2003) have shown that N_* is roughly equal to $4N_{\rm tr}$, where $N_{\rm tr}$ is the column density of the corona/chromosphere interface. (Although KBM used the Kramers bremsstrahlung cross-section $Q(\epsilon, E) \sim 1/\epsilon E$ to find $I(\epsilon)$, Equation [3] relates solely to electron dynamics and so is valid for *any* bremsstrahlung cross-section. To see this explicitly, one can include a cross-section multiplying factor $q(\epsilon, E)$ in the *outer* [bremsstrahlung-related] integrals in KBM without changing the inner [electron-dynamic related] expression.)

Having established the quantity $\bar{F}(E)$ purely from modelling of electron dynamics, one can then discuss its properties and compare them with those of the $\bar{F}(E)$ that is inferred from observational analysis (Equation [1]). For example, at small and large values of E/E_* the source-integrated spectrum (3) respectively approaches the limits

$$\bar{F}(E) \sim \frac{A\,C}{(\delta-1)K} E^{2-\delta}; \quad \bar{F}(E) \sim (1+\nu)\,\frac{A\,C}{(\delta-1)K} E^{2-\delta}. \quad (4)$$

The former is the result for a wholly *ionized* target, since low-energy electrons are mostly stopped in the ionized corona, while the latter is the result for a wholly *neutral* target, since the high-energy electrons are mostly stopped in the chromosphere. The source-integrated spectrum $A \int F(E,N)\,dN$ ($\propto \bar{F}[E]$) thus transitions from a power law of index $(\delta - 2)$ at low energies to a power law of the same slope, but larger by a factor $(1+\nu)$, at high energies, transitioning at $E \sim E_*$. Figure 1 shows the shape of $\bar{F}(E)$ for various δ, where a value $\nu = 1.8$ (cf. Brown et al. 1998) has been used. The spectrum flattens in the vicinity of E_*, so that the "local" electron spectral index $\delta_E(E) = -{\rm d}\log(\bar{F}(E))/{\rm d}\log E$ is reduced relative to the value in a uniform target. Figure 2 shows the difference between $\delta_E(E)$ and the uniform-target value $(\delta - 2)$ for various δ. The spectrum hardens by approximately half a power near E_*; the amount of the hardening and the energy at which the spectrum is hardest both increase with δ. Such a signature in $\delta_E(E)$ forms inferred from hard X-ray data could be indicative of a nonuniform target ionization, with a transition region column density $N_{\rm tr} \simeq (1/4) \times (E_*^2/2K)$, where E_* is the location of the minimum in δ_E. Such analysis could in principle determine the variation of $N_{\rm tr}$ throughout the duration of a flare for comparison with hydrodynamic models of atmospheric response to flare heating (cf. Kontar et al. 2003).

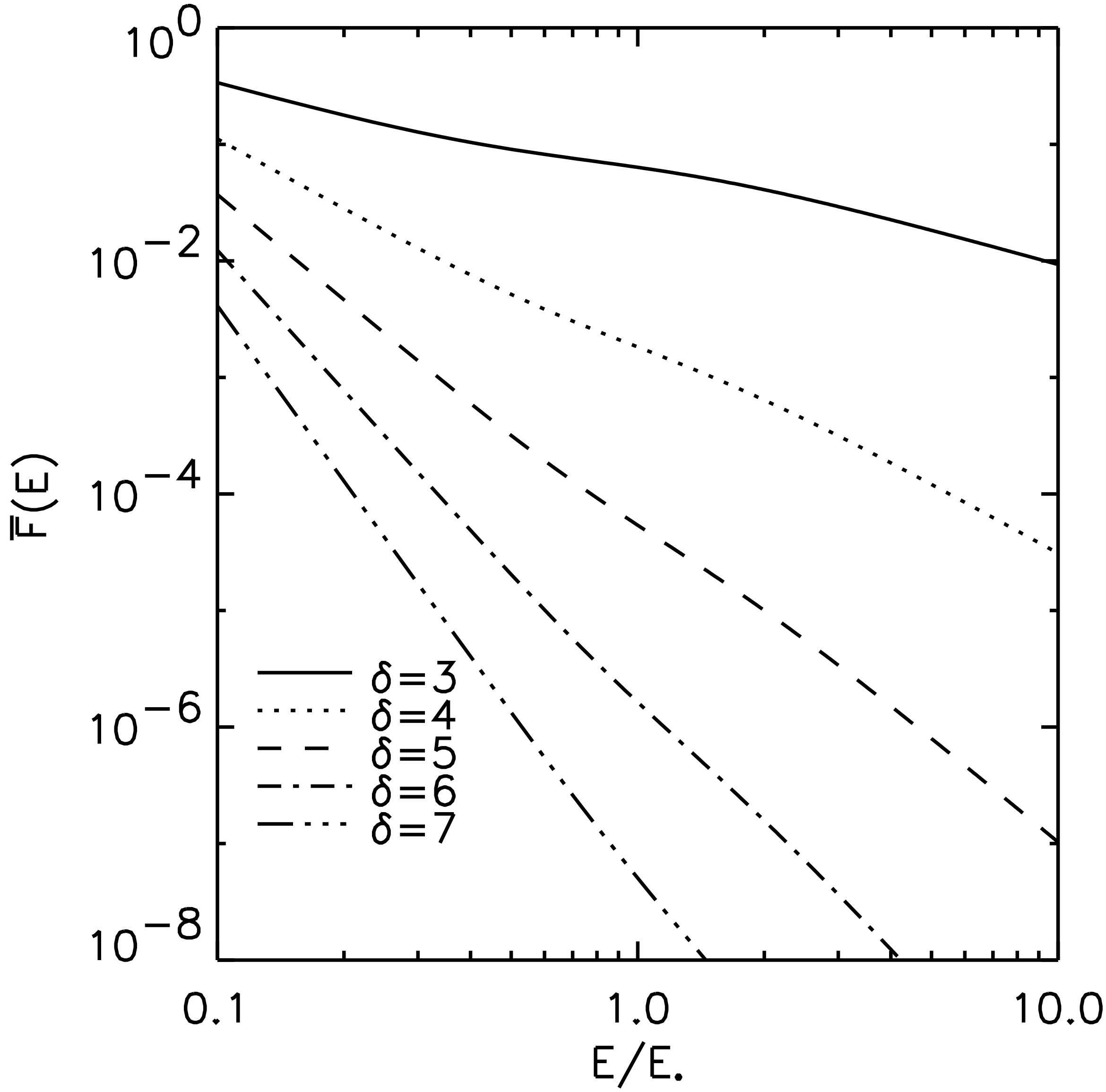

Fig. 1.— Form of $\bar{F}(E)$ for a power-law spectrum injected into a nonuniformly ionized target.

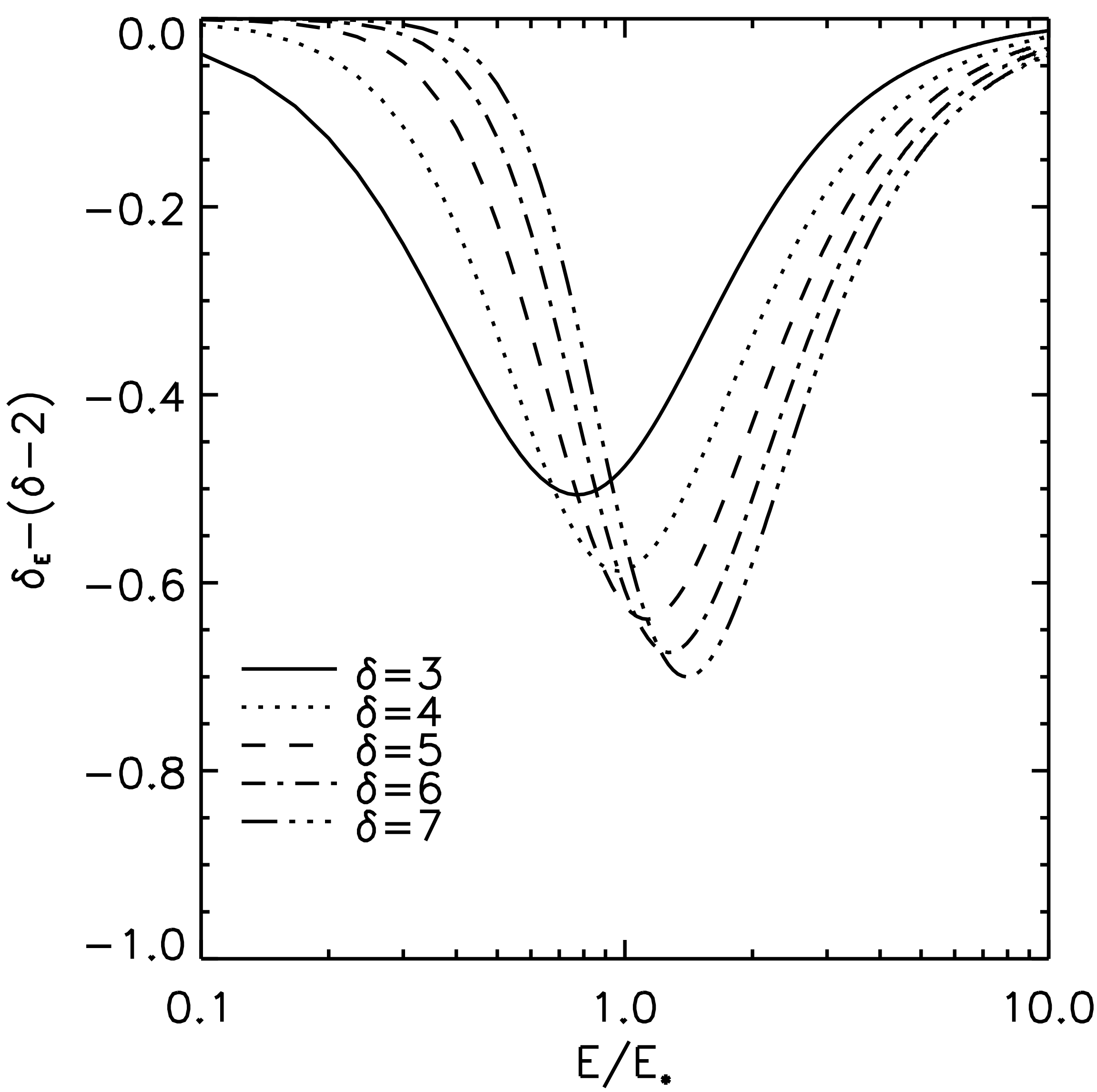

Fig. 2.— Variation of δ_E relative to its uniform-target value $(\delta - 2)$ as a function of energy for the mean target spectrum form [cf. Figure 1], for various values of the injected spectral index δ.

4. Constraints on The Mean Electron Spectrum in Various Source Models

Brown & Emslie (1988; hereafter BE) discussed analytic constraints on bremsstrahlung spectra produced by various types of source, such as a thick target and a thermal (but not necessarily isothermal) source. Their results were presented as constraints on *photon* spectra, using the Kramers cross-section. Here we recognize the important fact that these results are more properly expressed as constraints on the mean *electron* spectrum $\bar{F}(E)$, independent of $Q(\epsilon, E)$. For example, consistency of a $\bar{F}(E)$ with a collisional thick-target model requires (Equation [10] of BE)

$$\frac{d}{dE}\left[\frac{1}{E}\bar{F}(E)\right] < 0 \tag{5}$$

while for a *thermal* source (Equation [16] of BE),

$$(-1)^n \frac{d^n}{dE^n}\left[\frac{1}{E}\bar{F}(E)\right] > 0 \quad \forall n. \tag{6}$$

This "alternating signs of multiple derivatives" condition arises by repeated differentiation with respect to E of (Equation [13] of BE)

$$\frac{\bar{F}(E)}{E} \propto \int_0^\infty \frac{\xi(T)}{T^{3/2}}\, e^{-E/T}\, dT, \tag{7}$$

where T is the temperature (keV) and $\xi(T)$ is the differential emission measure. This shows that $\bar{F}(E)/E$ is the Laplace transform of the non-negative function $\hat{\xi}(s)$ (related to $\xi[T]$ by $\hat{\xi}[s] = \xi[1/s]/\sqrt{s}$), so that an $\bar{F}(E)$ is consistent with a thermal source if and only if its inverse Laplace transform is everywhere non-negative.

We can, for example, ask whether the $\bar{F}(E)$ corresponding to the injection of a power-law into a nonuniformly ionized target produces a photon spectrum that could be interpreted as thermal. Taking the inverse Laplace transform of the $\bar{F}(E)/E$ of Equation (3) gives, within a constant factor,

$$\hat{\xi}(s) = \frac{s^{\delta-2}}{\Gamma(\delta-1)} + \nu\, \frac{\sqrt{\pi}}{\Gamma\left(\frac{\delta-1}{2}\right)} \left(\frac{s}{2E_*}\right)^{\frac{\delta}{2}-1} J_{\frac{\delta}{2}-1}(E_* s), \tag{8}$$

where $J_n(x)$ is the Bessel function of order n. For $\nu = 1.8$, all values of $\delta > 2$ yield positive definite functions $\hat{\xi}(s)$. Thus quite diverse (and not intrinsically thermal) forms of $\bar{F}(E)$ can be mimicked by thermal sources. However, by no means can *all* forms of $\bar{F}(E)$ be produced by a thermal source (cf. BE). In particular, $\hat{\xi}(s)$ is ≥ 0 only for sufficiently small ν; for larger ν the oscillatory term in (8)would be magnified to yield (unphysical) regions of negative $\xi(T)$.

5. Summary

The primary purpose of this communication is to establish a natural fiducial "middle ground" for comparison of hard X-ray data with models of electron acceleration and transport in solar flares. We have shown that the mean electron flux spectrum $\bar{F}(E)$ is the natural candidate for this middle ground, separating cleanly the effects of radiation physics and electron dynamics. We have presented criteria that $\bar{F}(E)$ must satisfy in certain source models and easily-identifiable characteristics of $\bar{F}(E)$ forms in others. In companion papers (Holman et al. 2003, Piana et al. 2003), forms of $\bar{F}(E)$ deduced from RHESSI observations of the July 23, 2002 solar flare are presented and briefly discussed.

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