Solar Flare Hard X-ray Spectra Possibly Inconsistent with the Collisional Thick Target Model

Eduard P. Kontar and John C. Brown

Department of Physics & Astronomy, University of Glasgow, G12 8QQ, United Kingdom

Abstract

Recent progress in solar Hard X-ray (HXR) observations with RHESSI data and methods for spectral inversion allow us to study model-independent mean electron flux spectra in solar flares. We report several hard X-ray events observed by RHESSI in which the photon spectra $I(\epsilon)$ are such that the inferred source mean electron spectra are not consistent with the standard model of collisional transport in solar flares. The observed photon spectra are so flat locally that the recovered mean electron flux spectra show a dip around 17-31 keV. While we note that alternative explanations, unrelated to electron transport, have not been ruled out, we focus on the physical implications of this tentative result for the collisional thick-target model.

Key words: Sun, Solar Flares, Hard X-rays, Energetic Particles

1 Introduction

It has long been recognized (Brown, 1971; Brown and Emslie, 1988; Johns and Lin, 1992) that even spatially integrated spectra $I(\epsilon)$ of flare hard X-ray (HXR) collisional bremsstrahlung bursts carry crucial information on flare electron acceleration, transport, and energy budget. Even the absence of spectral features, such as in an (energy-scale-free) pure power-law, is likely to indicate stochastic multi-scale processes. On the other hand, any features which exist in the acceleration spectrum are smeared out in the propagation and radiation process so that any features detected in $I(\epsilon)$ have potentially very

Email address: eduard@astro.gla.ac.uk, john@astro.gla.ac.uk (Eduard P. Kontar and John C. Brown).

strong implications for the propagation (Kontar et al, 2003) and acceleration processes, and for the electron energy budget in terms of any low energy cut-off.

The large data base of high resolution spectra from the RHESSI mission (Lin et al, 2002) has made possible a thorough search for such special HXR spectral diagnostic properties. Care has to be taken, however, not to misinterpret features as entirely real solar ones, such as the 50 keV electron spectral feature in the July 23, 2002 event (Piana et al, 2003). This can be attributed partly to pulse pile-up at the high count rate in this event. Two approaches are used in interpreting $I(\epsilon)$ data. One is to adopt a parametric form for the mean radiation source electron spectrum (or for the electron 'injection' spectrum) and predict $I(\epsilon)$, using an accurate bremsstrahlung cross-section $Q(\epsilon, E)$, and search for the best fit in the model parameter space. This is commonly done using a source-mean flux spectra $\overline{F}(E)$ containing an isothermal Maxwellian, $F_T(E)$, defined by an emission measure EM and temperature T, plus a non-thermal $F_{pow}(E)$ parameterized as a single or double power-law with a sharp low energy cut-off, E_c (Holman et al, 2003). Such best fits tend to yield an $E_c \sim 10 - 30$ keV, well above $kT \sim 1 - 3$ keV, but with EM large enough that E_c is 'buried' in the $F_T(E)$ 'tail' and it is impossible to be sure whether $F_{pow}(E)$ has an actual cut off or blends smoothly with $F_T(E)$, especially if there is some spread in T. Recently, Schwartz et al (2003) have reported, for the August 20, 2002 flare that a best fit requires an $E_c \approx 30$ keV in $F_{pow}(E)$ seen clearly above the isothermal best fit $F_T(E)$. To evaluate whether such a real feature/cut-off is demanded by the data, as opposed to simply being compatible with it, it is essential to adopt a non-parametric approach to interpreting $I(\epsilon)$ - i.e. to infer from $I(\epsilon)$ what range of functions $\overline{F}(E)$ allows a statistically acceptable fit to $I(\epsilon)$. This inverse/inferential approach has been adopted, developed and applied to data by a number of authors (Craig and Brown 1986, Johns and Lin, 1992; Thomson et al., 1992; Piana et al, 2003; Kontar et al, 2004; Kontar et al, 2005).

In this paper we apply one of the best available inversion algorithms of Kontar et al (2004) to several events we have found in the RHESSI database to show real gaps or dips in $\overline{F}(E)$ probably demanded by, and not just consistent with, the data. The proposed implications of these results for flare electron acceleration, propagation, and energy budget are briefly discussed.

2 Bremsstrahlung Source Models and Constraints on the Emission Spectra

2.1 *Mean source electron spectra $\overline{F}(E)$*

In this section we generalize to arbitrary cross-section $Q(\epsilon, E)$ discussions of bremsstrahlung spectrum constraints for models discussed earlier by various authors.

Following Brown (1971) and Brown, Emslie and Kontar (2003) we emphasize that the electron distribution function which can be inferred from $I(\epsilon)$ without source model assumptions (apart from optical thinness and isotropy) is the density weighted mean radiation source electron flux spectrum $\overline{F}(E)$ (electrons cm^{-2} s^{-1} keV^{-1}) defined by

$$\bar{F}(E) = \frac{1}{\bar{n}V} \int\limits_V F(E, \mathbf{r})\, n(\mathbf{r})\, dV. \tag{1}$$

where $F(E, \mathbf{r})$, $n(\mathbf{r})$ are the local electron flux spectrum and source proton density at position $\mathbf{r}$ in radiating volume V with mean target proton density $\bar{n} = V^{-1} \int n(\mathbf{r})\, dV$. $\overline{F}(E)$ uniquely defines the bremsstrahlung spectrum at the earth by

$$I(\epsilon) = \frac{1}{4\pi R^2}\, \bar{n}V \int\limits_{\epsilon}^{\infty} \bar{F}(E)\, Q(\epsilon, E)\, dE, \tag{2}$$

where $Q(\epsilon, E)$ is the cross-section differential in ϵ (Haug, 1997). In general $Q(\epsilon, E)$ should include angle averaging to allow for anisotropy of the electron distribution function. The only correction we do not discuss explicitly here is for photospheric back scatter, though it might be important for our results.

The general approach to Equation (2) is to treat it as an integral equation to reconstruct $\overline{F}(E)$ from $I(\epsilon)$ by deconvolution through $Q(\epsilon, E)$ using regularization techniques e.g. (Craig and Brown, 1986; Kontar et al, 2004). Limitations can be placed on $I(\epsilon)$ for a physically acceptable solution of (2) to exist. In particular, generalizing (Brown and Emslie, 1988),

$$\frac{d[\epsilon I(\epsilon)]}{d\epsilon} = -\bar{n}V \left. \frac{\bar{F}(E)\epsilon Q(\epsilon, E)}{4\pi R^2} \right|_{E=\epsilon} + \\ + \frac{\bar{n}V}{4\pi R^2} \int\limits_{\epsilon}^{\infty} \bar{F}(E)\, \frac{\partial[\epsilon Q(\epsilon, E)]}{\partial \epsilon}\, \mathrm{d}E. \tag{3}$$

Since any physical $\overline{F}(E)$ should be nonnegative and since $\partial(Q(\epsilon, E)\epsilon)/\partial\epsilon < 0$ $\forall\epsilon > 0$ for the relativistic cross-section of Haug, (1997), we immediately have that $d(I(\epsilon)\epsilon)/d\epsilon < 0$, $\forall\epsilon > 0$. It is useful to consider the shape of $I(\epsilon)$ in terms of the local spectral index $\gamma(\epsilon)$ defined by

$$\gamma(\epsilon) \equiv -\frac{\epsilon}{I(\epsilon)}\frac{dI(\epsilon)}{d\epsilon}, \tag{4}$$

(note that definition (4) does not correspond to $I(\epsilon) \sim \epsilon^{-\gamma(\epsilon)}$ except for constant γ (Conway et al, 2003)). Then in terms of the spectral index the condition $d(I(\epsilon)\epsilon)/d\epsilon < 0$ can be expressed

$$\gamma(\epsilon) > 1, \quad \forall\epsilon \tag{5}$$

i.e. any bremsstrahlung emission spectrum $I(\epsilon)$ should decrease with a logarithmic gradient larger than 1. If the condition (5) is violated then the spectrum $I(\epsilon)$ is not from an optically thin bremsstrahlung source.

To date no HXR spectrum from a solar flare violating Condition (5) has ever been reported, so there is no challenge to the belief that flare HXR burst spectra are consistent with optically thin bremsstrahlung. However, more stringent spectral compatibility conditions apply to specific models of how $\bar{F}(E)$ is formed from an injection spectrum $F_0(E_0)$.

2.2 Thick target model

A *thick target* is a source in which electrons injected into the source decelerate to rest under the action of energy loss processes, within the observed integration time and volume. The terminology 'thick' is similar to optical depth, a thick target being a medium where plasma electron column density is sufficient to stop energetic electrons, or, equally, the mean free path of the fast electrons is much less than the system length. In other words, for energy loss cross section $Q_E = -d\log(E)/dN$, $NQ_E \gg 1$ within the column density N of V at all E of interest. The resulting mean electron flux spectrum is then the density-weighted average of the flux spectrum of the electrons along their energy loss path z, as governed by Q_E for all the energy loss processes involved.

The assumptions in inferring $F_0(E_0)$ from the total $I(\epsilon)$ are that : (a) the bremsstrahlung emission from acceleration region can be neglected, and (b) physically distinct 'acceleration' and 'target' (deceleration) regions exist.

Let us consider the motion for an electron in the target from an injection spectrum $F_0(E_0)$ $(E(z = 0) = E_0)$ for a given background plasma density profile $n(z)$, and introduce the column depth $N(z)$ of the plasma so that $n(z) = dN/dz$. It is important to note that z is an electron path. Therefore, a column depth can be also defined as $dN(z) = n(z)v(z)dt$. Comparing with the original thick target model (Brown, 1971) where only collisional losses have been taken into account we get, for that case

$$\frac{dE}{dN}_{coll} = -K/E \tag{6}$$

where $K = 2\pi e^4\Lambda$, e is the electron charge,and $\Lambda \approx 20 - 25$ is the Coulomb logarithm.

Assuming that we can find the solution of the equation of motion of an electron in the form $E = E(E_0, N)$ we can write down the mean electron flux in the following form

$$\bar{F}(E) = \frac{A}{\bar{n}V} \int\limits_0^\infty F(E(E_0, N))\, dN. \tag{7}$$

where A is the injected area. The evolving electron spectrum $F(E(E_0, N))$ can then be expressed in terms of the initial or injected electron spectrum using the continuity equation $F(E(E_0, N))dE = F_0(E_0)dE_0$. After the change of variables $N \to E_0$ in (7) one has for the mean electron spectrum with arbitrary energy losses.

$$\bar{F}(E) = -\frac{A}{\bar{n}V} \int\limits_E^{E_0(N=\infty,E)} F_0(E_0) \left(\frac{dE}{dN}\right)^{-1} dE_0. \tag{8}$$

(The mean electron flux given by (8) makes use of deterministic electron propagation ignoring collective and diffusion effects). In previous papers (Brown and MacKinnon, 1985) it was also assumed that the upper limit in the integral (8) is infinity [1], e.g. $E_0(N \to \infty, E) = \infty$, which is true for purely collisional losses. Equations (6) and (8) become

$$\bar{F}(E) = \frac{AE}{K\bar{n}V} \int\limits_E^\infty F_0(E_0)dE_0. \tag{9}$$

[1] Actually $E_0(N \to \infty, E) = \infty$ for any non-collisional losses described by the equation of the type $\frac{dE}{dN} = -\varphi(E)$

2.3 Purely collisional thick target

We now consider the case of a purely collisional thick target (9), when the injection spectrum takes the form

$$F_0(E_0) = -\frac{K\bar{n}V}{A}\frac{d}{dE}\frac{\bar{F}(E)}{E}\bigg|_{E=E_0} = \\ = K\frac{\bar{n}V}{A}\frac{\bar{F}}{E_0^2}\left(1+\delta(E)\right)_{E=E_0} \quad (10)$$

where $\delta(E) = -d\ln\bar{F}/d\ln E$ is the spectral index of the density weighted mean electron flux distribution. From equation (10) it follows that, for a photon spectrum $I(\epsilon)$ to be produced by a purely collisional thick target with a physical (nonnegative) injection spectrum $F_0(E_0) \geq 0$, the mean electron flux $\bar{F}(E)$ derived from the photon spectrum $I(\epsilon)$ must have a logarithmic slope

$$\delta(E) \geq -1 \qquad \forall E \neq 0 \quad (11)$$

(or $d\log\bar{F}/d\log E \leq +1$).

In the next section we report RHESSI $I(\epsilon)$ data in which the above condition appears to be violated, and a purely collisional thick target interpretation is impossible.

Equation (9) also explicitly says that if the injected spectrum $F_0(E_0)$ has a low energy cut-off at energy E_0, then the purely collisional thick target mean spectrum $\bar{F}(E)$ can have a spectral index $\delta(E) = -1$, for $E < E_0$ i.e. a logarithmic slope of $\geq +1$) which is the lowest δ value that can be explained by a collisional thick target. However, if the spectral index at some point is less than -1, then condition (11) is violated, and non-collisional models must be involved.

3 RHESSI observations

There are flares observed by RHESSI (Lin et al, 2002) with unusually flat photon spectra resulting in peculiar mean electron distribution functions $\bar{F}(E)$. Unlike the July 23, 2002 flare (Piana et al, 2003), in these flares the count rate in the front segments of the RHESSI detectors were too low for pulse pile-up (Smith et al, 2002) to be important. Forward fits applied to this data show that the X-ray spectra can be best fitted by a broken power-law with a low energy cutoff in the range between 15 and 35 keV. This suggests that there are no injected nonthermal electrons in the range below a few tens of keV. To verify this we used a recently developed regularization algorithm (Kontar et al. 2004) to infer model - free electron spectra for these events. The mean electron flux spectrum inferred shows a clear dip at energies between 15 and 35 keV. A similar feature, though less clear and at the energies around 50keV, has been recovered by Piana et al (2003). The main results are presented in Fig 1-3.

3.1 Inversion of photon spectra

The observed spectrum of the flare is the convolution of the cross-section and mean electron flux. The problem of inferring the mean electron spectrum is ill-posed (Craig and Brown, 1986). Thus, one has to avoid unphysical behaviour in $\bar{F}(E)$ using some constraints. This process is called regularization.

To infer the mean electron spectrum we used the following recently developed regularization algorithm of Kontar et al, (2004)

$$\mathcal{L}(\overline{\mathbf{F}}) = \|\mathbf{A}\overline{\mathbf{F}} - \mathbf{I}\|^{\mathbf{2}} + \lambda\|\mathbf{L}\overline{\mathbf{F}}\|^{\mathbf{2}}, \quad \mathcal{L}(\overline{\mathbf{F}}) = \min \tag{12}$$

where λ is a regularization parameter, $\mathbf{L}$ is a regularizing first order derivative operator, and $\mathbf{A}$ is the matrix representation of the cross-section operator A in

$$(A\overline{F})(\epsilon) \equiv \frac{1}{4\pi R^2}\,\overline{n}\,V \int\limits_{\epsilon}^{\infty} \overline{F}(E)\,Q(\epsilon, E)\,dE, \tag{13}$$

i.e. comprises binned integral components $\int_{\Delta E_j} Q(\epsilon_i, E)\mathrm{d}E$. For our calculations we used the Haug (1997) cross-section.

One should note that if we try to infer the electron spectra without additional constraints, e.g. setting to zero the penalty term in Eq. 12, the resulting $\bar{F}(E)$ would be a violently oscillating function. Such unphysical results come from the ill-posedness of the problem, since small noise perturbations in $I(\epsilon)$ will be strongly amplified.

3.2 Mean Electron Spectrum and Spectral Index

For our analysis we have selected three flares with unusually flat local spectra. The data reduction has been performed in the same way as in Kontar et al (2005). Then the regularization algorithm has been applied to find the mean

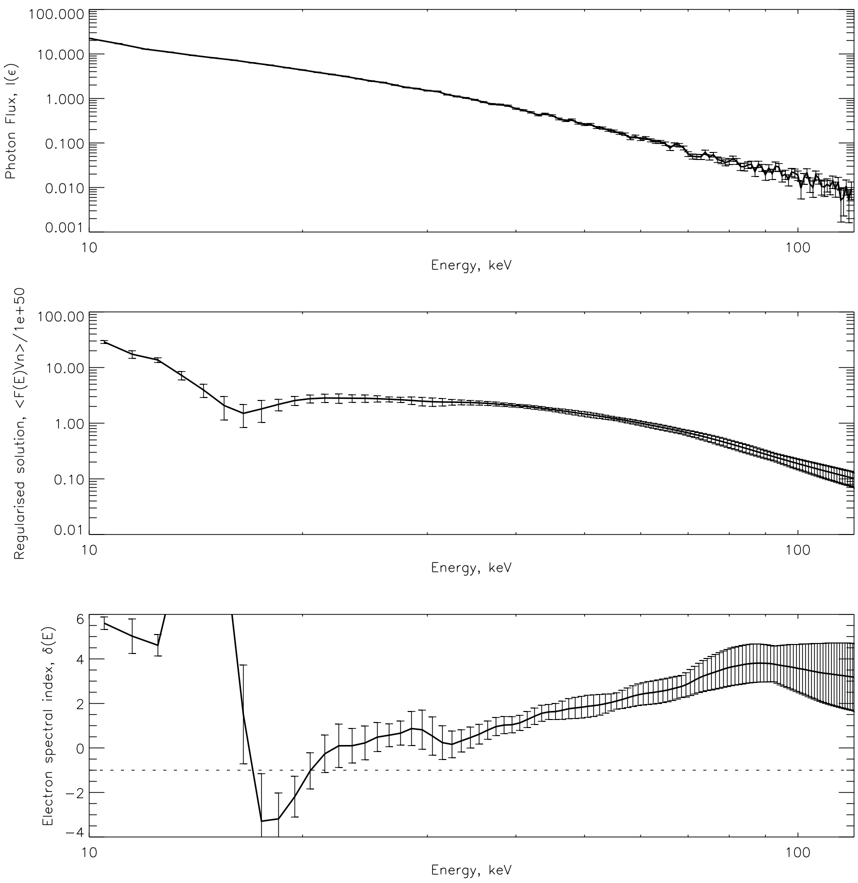

Fig. 1. Spatially integrated photon flux $I(\epsilon)$ (upper panel), mean electron flux $\overline{F}(E)$ and electron spectral index $\delta(E)$ (lower panel) for 25 April 2002 05:55 UT flare using first order regularization. The photon spectrum has been accumulated over the impulsive phase of the flare.

electron flux and its spectral index. Figures 1-3 show flat mean electron flux spectra with a positive derivative in the range 15-35 keV. The depth of the dip and the resulting electron spectral index is different from flare to flare, though qualitatively similar. The April 25 event (Figure 1) has a dip at 19 keV but the error bars do not allow us to conclude that this dip is real.

The confidence intervals on the regularized solution and spectral index (Figures 1-3) have been calculated as a maximum deviation from the solution in a set of 30 random realizations of photon data within a $\pm 1\sigma$ range. The distribution of errors has been discussed in Piana et al 2003; Kontar et al, 2004. To allow for instrumental uncertainties, 1σ was taken to be not less than 3% of the photon flux, and thus can be treated as an upper estimate of the error.

The August 2002 (Figure 2) flare shows an extremely clear gap around 20 keV in the mean electron flux. Forward-fit to the data shows an unambiguous low energy cut-off at ~ 31 keV (Kasparova et al, 2005). To verify the reality

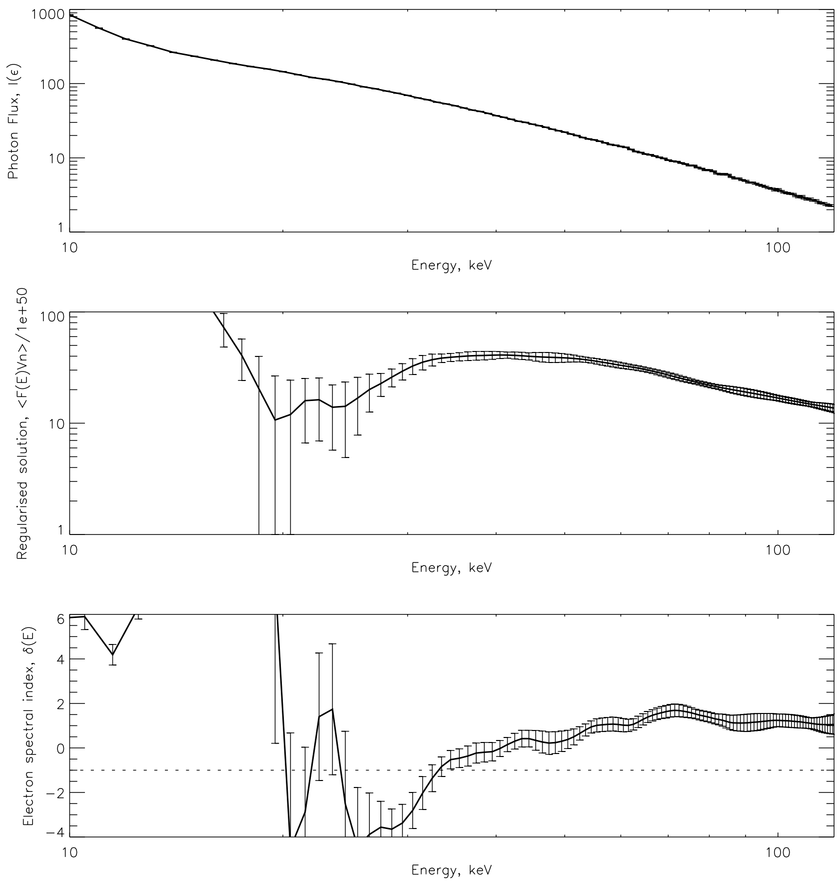

Fig. 2. The same as in Fig 1 for 20 August 2002 08:24 UT flare.

of this feature, we used different orders of regularization for the inversion of the corresponding flare X-ray spectra. All zero, first, and second order regularization methods produce the dip (Figure 2).

Figure 3 also shows that at least three successive points have δ less than -1 with 1σ confidence and 4 points less than zero. The former suggests that the thick-target model is unacceptable while the latter indicates a low energy cut-off in the injected electron spectrum. Though we have used the 1σ criterion, the result is much more significant than that 65% of a single 1σ deviation since there are three successive such deviations. While a more rigorous statistical analysis is necessary, this should be significant at roughly the level of $1-(1-0.65)^3$ or about 97% which makes it highly suggestive at least.

4 Discussion and conclusions

Our analysis of the observed photon spectra shows features that have not been observed before. Johns and Lin (1992) reported the downturn of the derived

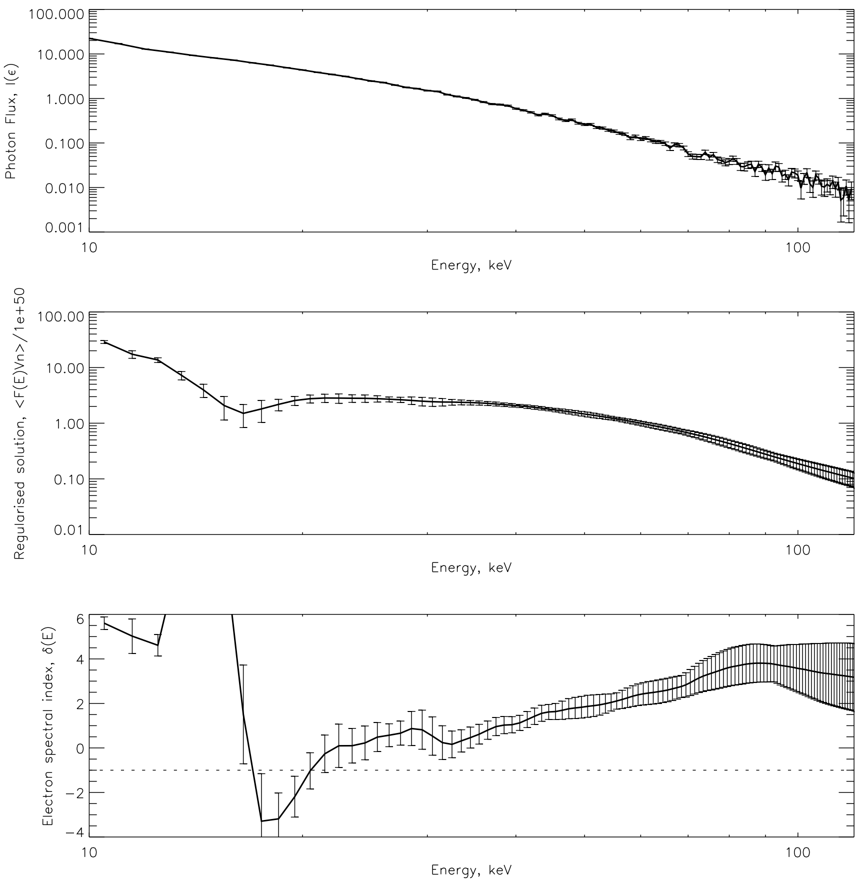

Fig. 3. The same as in Fig 1 for 17 Sept 2002 05:55 UT flare.

electron spectrum for earlier balloon data, though the errors were too big to be conclusive. In the results presented in this paper we are much more confident that a dip in the electron distribution is required by the photon spectra, though we caution that the photon spectrum itself is subject to additional corrections for photospheric albedo, directivity, and instrument effects (Smith et al, 2002).

One of the important implications of our result if upheld is related to the accelerated spectrum $F_0(E_0)$. As can be seen from the results, the photon spectra require a low energy cut-off in the injected spectra $F_0(E_0)$ to be produced, e.g. the absence of electrons in the range below 20-30 keV. This is quite a strong requirement on acceleration models, that often produce extended power law spectra above thermal energies of a few keV. The cut-off poses physical challenges to electron acceleration and transport since an electron beam with a velocity distribution function such that $f(v) \sim \bar{F}(v^2)$ should be unstable to the generation of plasma waves if $df(v)/dv > 0$.

We stress that, if the existence of a low energy cut-off in the injected electron spectrum $F_0(E_0)$ is confirmed, its implications for total flare electron power, acceleration and propagation are profound. Moreover, the evidence of a steep turn over in $\bar{F}(E)$ would reject the long-standing collisional transport thick-target model of flare electron propagation (Brown, 1971) where the accelerated electron spectrum $F_0(E_0)$ is modified solely by Coulomb collisions along the electron path.

Acknowledgements We are grateful for the support of a PPARC Grant and to A.G. Emslie, C. Johns-Krull, J. Kasparova, R.P. Lin, A.M. Massone, M.Piana, and R.A. Schwartz for numerous stimulating discussions.

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