STATISTICAL ANALYSIS OF LANGMUIR WAVES ASSOCIATED WITH TYPE III SOLAR RADIO BURSTS

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l'Observatoire de Paris

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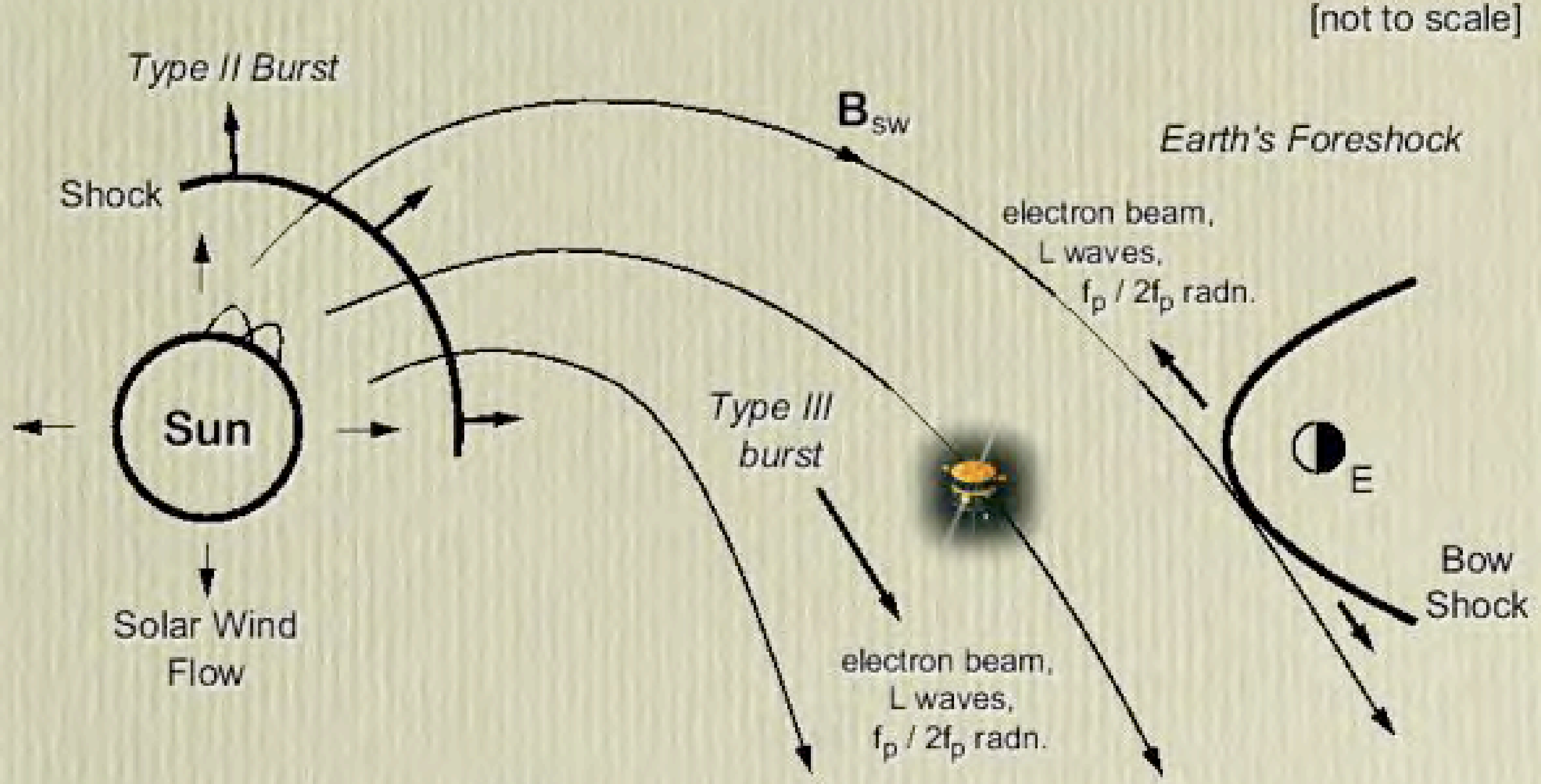
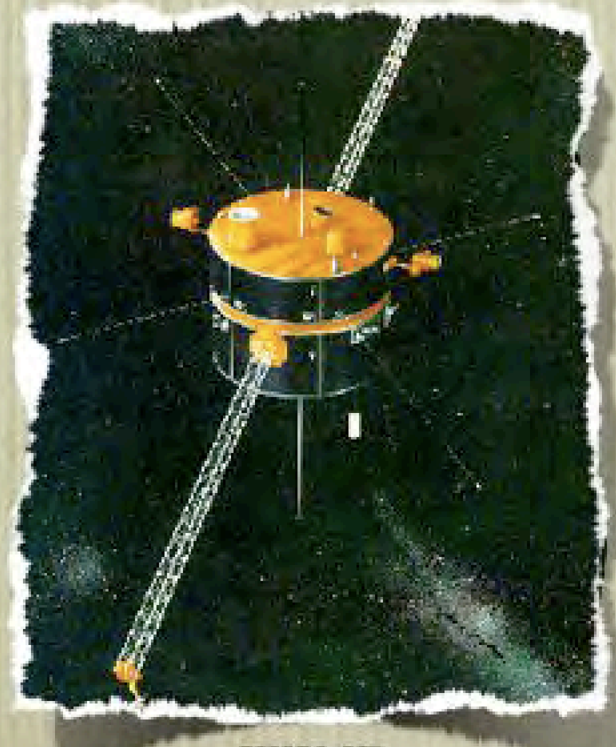
Introduction

Solar flare (Solar X-Ray Imager GOES-13).





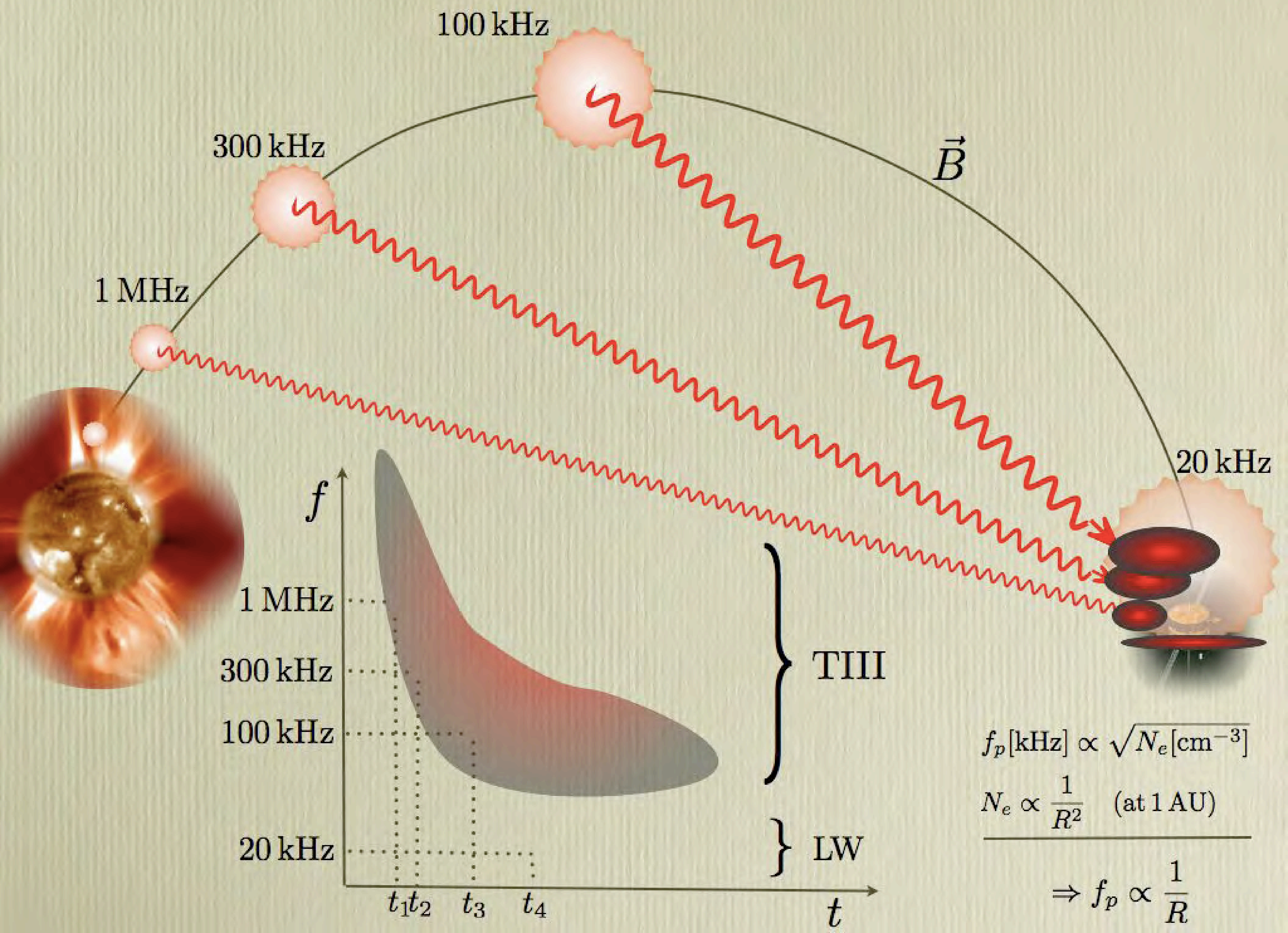
WIND

- Interaction between plasma (solar wind) and electron beam produces Langmuir waves (LW);

- LW convert into type III bursts (we observe them)

Q: What are characteristics of LW? How LW and TIII are related?

NOW, only part of the Q: Do the LW satisfy SGT?

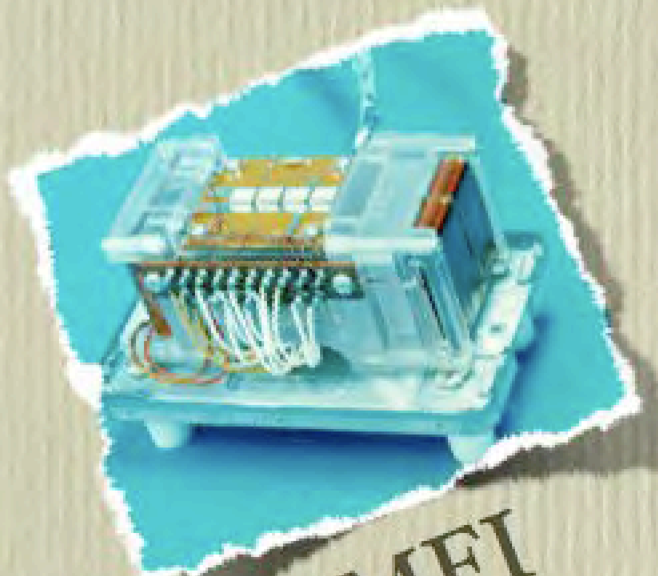


100 kHz
300 kHz
1 MHz
$\vec{B}$
20 kHz
f
1 MHz
300 kHz
100 kHz
20 kHz
$t_1 t_2$ t_3 t_4
t
TIII
LW
$f_p[\mathrm{kHz}] \propto \sqrt{N_e[\mathrm{cm}^{-3}]}$
$N_e \propto \frac{1}{R^2}$ (at 1 AU)
$\Rightarrow f_p \propto \frac{1}{R}$

In situ measurements

Experiments onboard WIND spacecraft:

WAVES

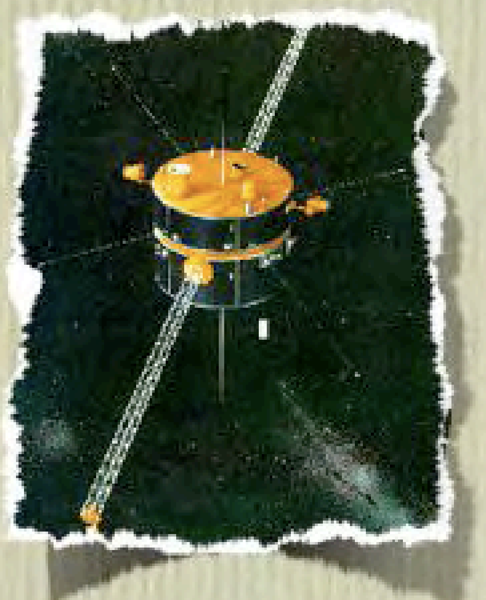


MFI

3DP

SWE

Observations

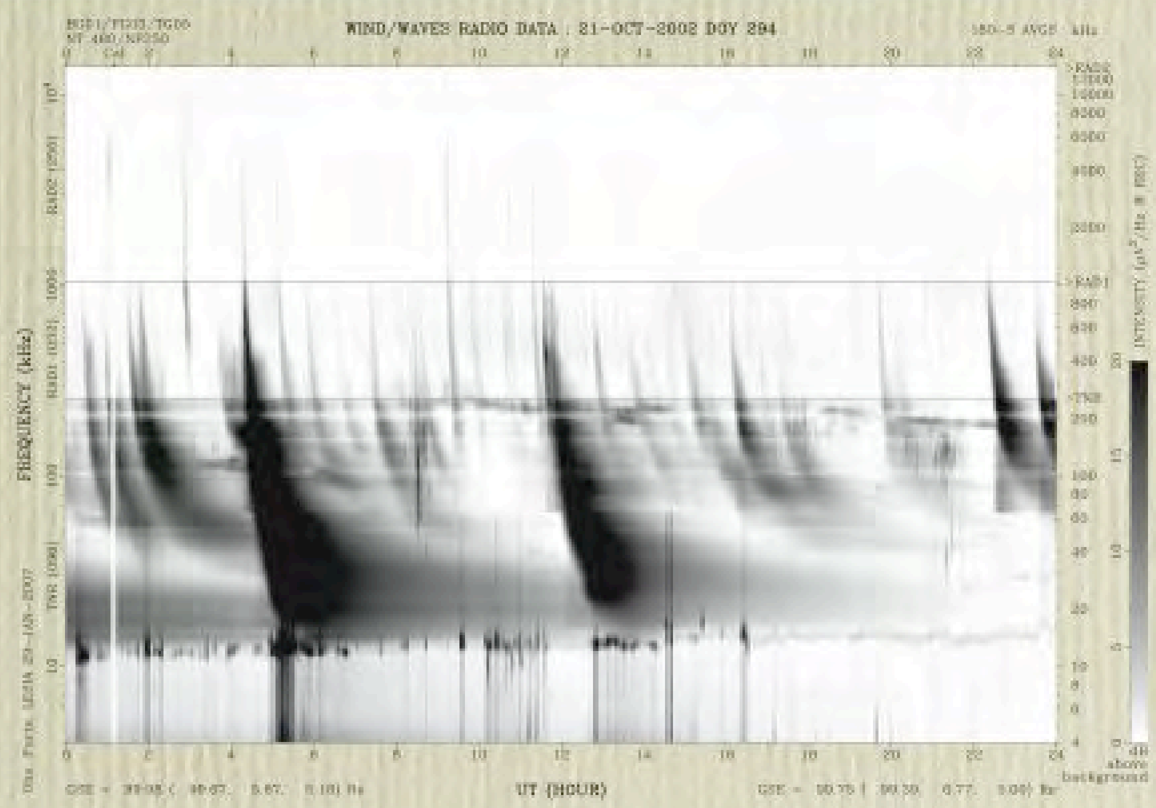


+

Wavelet-like analysis
using digital filters
equivalent to
Fourier Transformation*

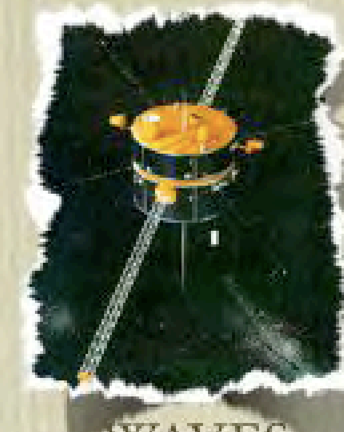
⇒

⇒

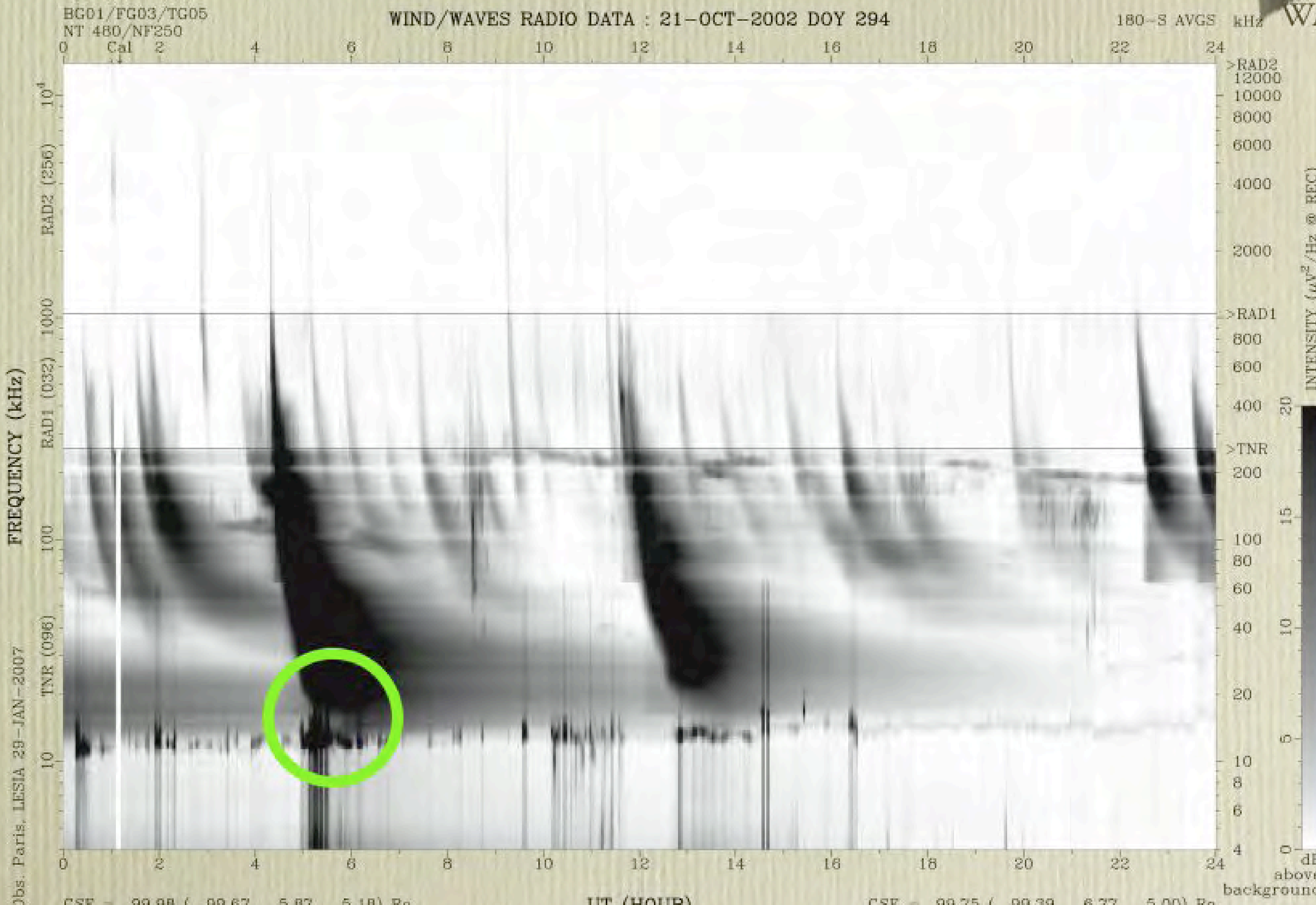


*Bougeret, J-L. et al : *WAVES: the radio and plasma wave investigation on the Wind spacecraft*, 1995, Sp. Sci. Rev., **71**, 231

Observations



WAVES



Stochastic growth theory (SGT)

- Electron beam – an unstable distribution of particles:
  interacts with its driven waves; surrounding environment is inhomogeneous plasma; it evolves to a state in which the particle distribution fluctuates STOCHASTICALLY about a state close to marginal stability; the fluctuations in the distribution drive waves so that wave gain is a STOCHASTIC variable!

$$G = 2 \log\left(\frac{E}{E_0}\right)$$

- Observed electric field, E, is a consequence of a large number of independent amplifications & dampings:

$$\log E = \log E_0 + \sum_{i=1}^{N} G_i \qquad N \gg 1$$

Then, the central limit theorem can be applied to the probability distribution of log E which is thus normal distribution*.

* Robinson, P. A. : *Stochastic-Growth Theory of Langmuir growth-rate fluctuations in type III solar radio sources,* 1993, Solar Physics, **146**, 357

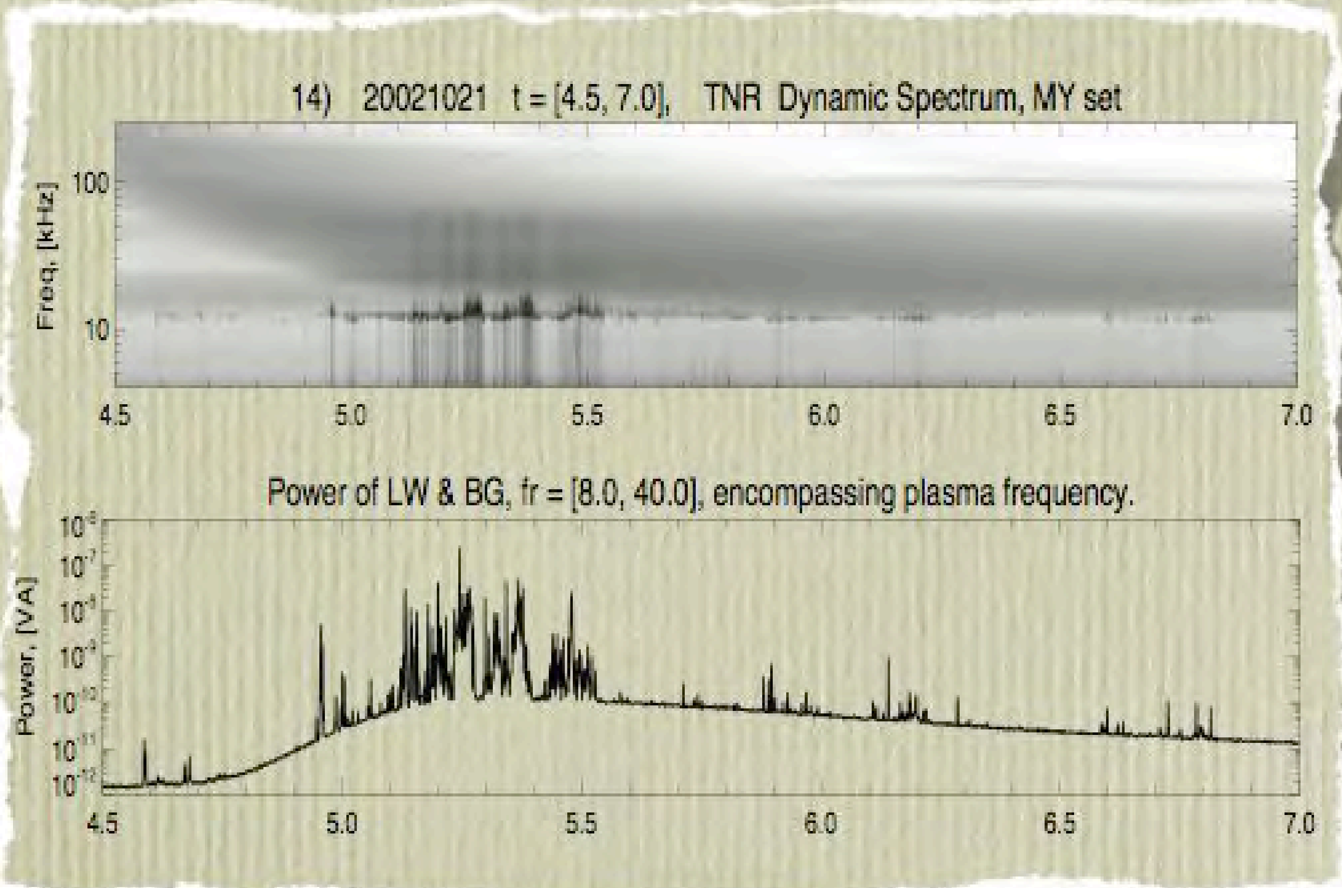
Q: Do the LW satisfied SGT?

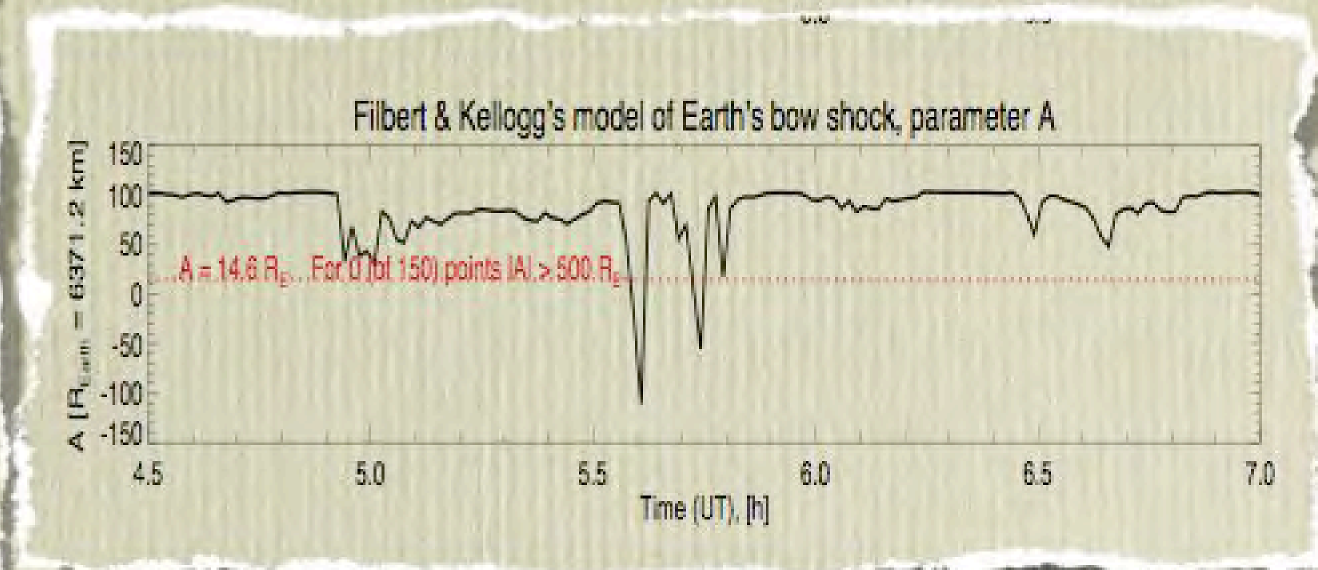
- In order to check that, we have undertaken the following steps:
  - Integrate power spectral density (S_{LW}) of LW through a narrow interval of frequencies around the local plasma frequency:

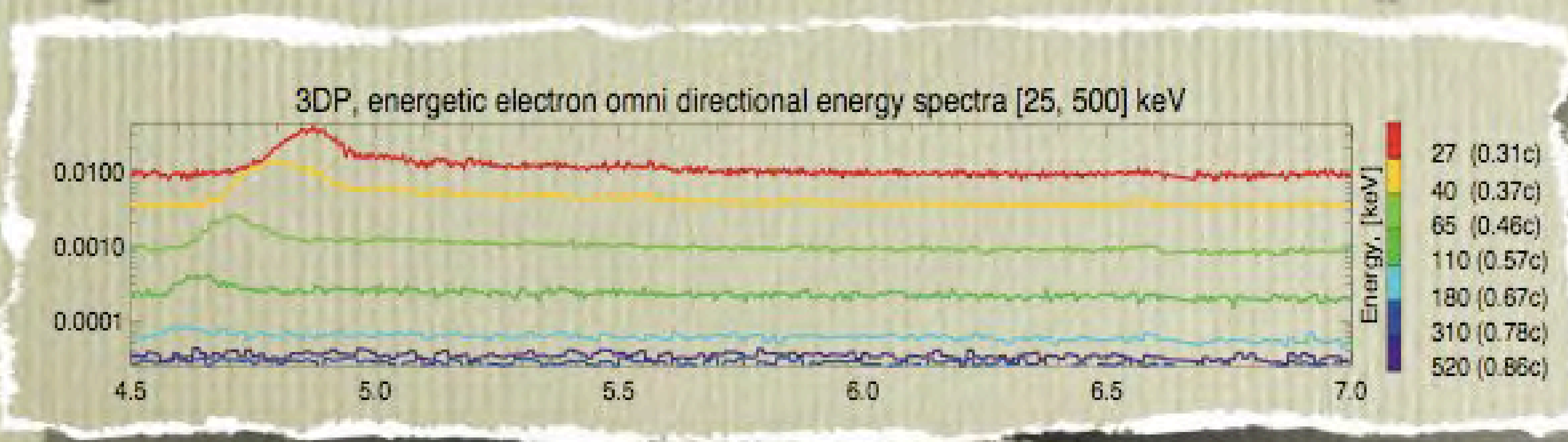
$$S_{LW,t} = \int_{f_1}^{f_2} S_t \, \mathrm{d}f, \qquad f_1 < f_p < f_2$$

  - Remove BG consisting of type III bursts Galactic BG and TN.
  - Fit with a normal probability distribution.

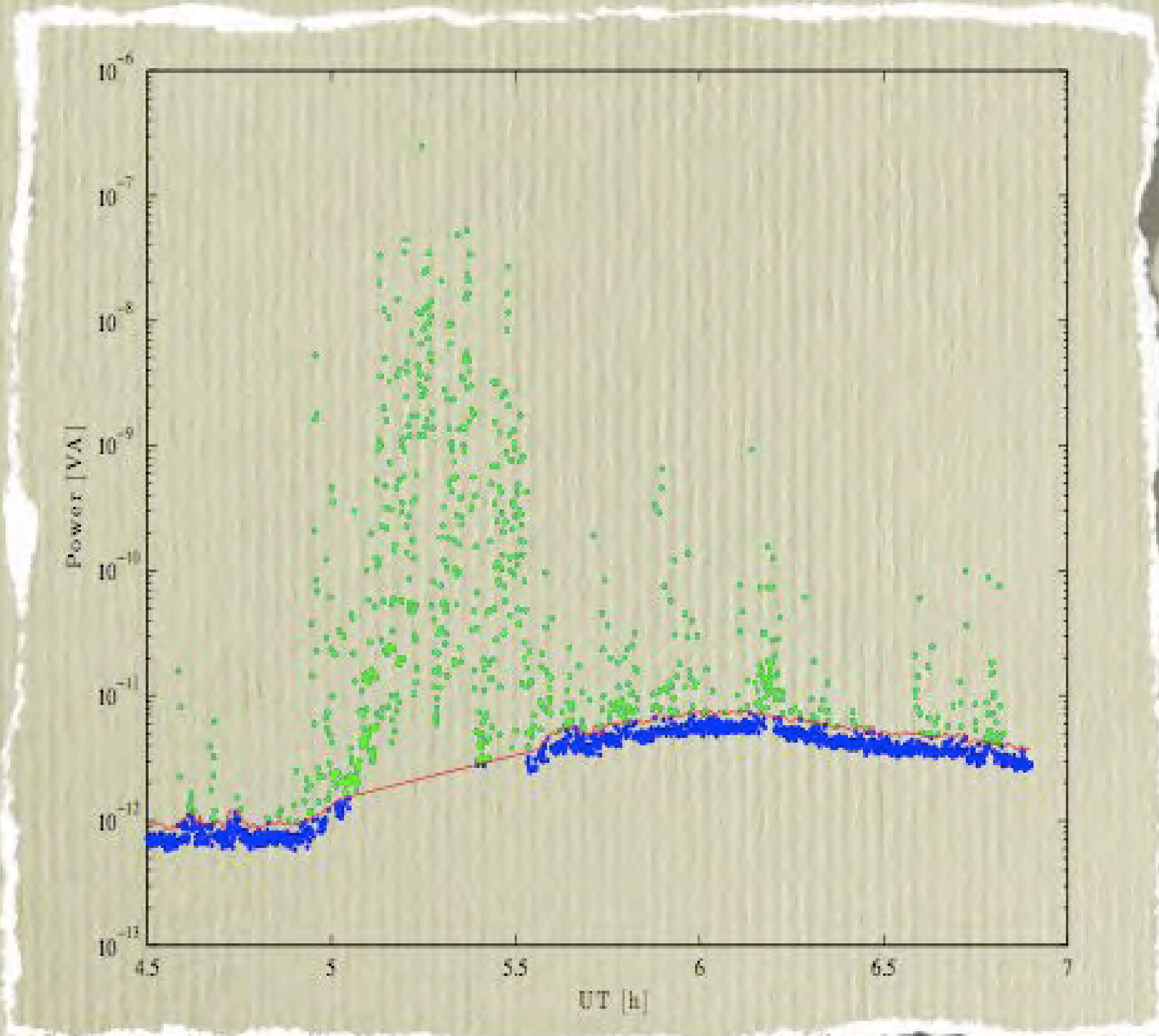
Sample events selection

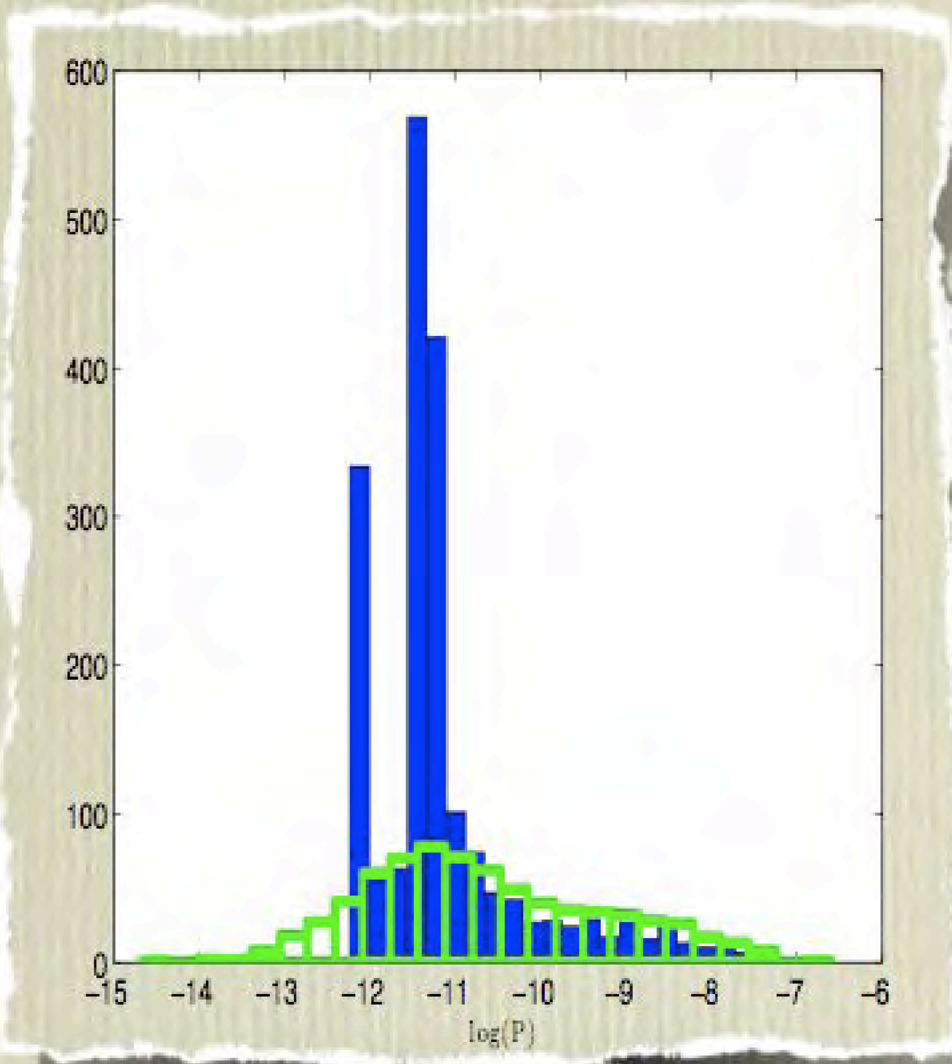






LW power





Pearson's system*

- First derivative of distribution:

$$\frac{1}{f(x)}\frac{\mathrm{d}f(x)}{\mathrm{d}x} = -\frac{a+x}{c_0 + c_1 x + c_2 x^2}$$

- Skewness (β_1)

$$\beta_1^2 = \frac{\mu_3^2}{\mu_2^3}$$

- Kurtosis (β_2)

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

Using only two parameters, square of skewness β_1 and kurtosis β_2 obtained from observations, we find that our 36 events belong to only 3 types of Pearson's distribution.

*Pearson, K.: 1895, *Contributions to the Mathematical Theory of Evolution. II. Skew Variation in Homogeneous Material.* Philosophical Transactions of the Royal Society of London, **186**, 343 – 414

Method of moments

$$c_0 = (4\beta_2 - 3\beta_1)(10\beta_2 - 12\beta_1 - 18)^{-1}\mu_2$$

$$a = c_1 = \sqrt{\beta_1}(\beta_2 + 3)(10\beta_2 - 12\beta_1 - 18)^{-1}\sqrt{\mu_2}$$

$$c_2 = (2\beta_2 - 3\beta_1 - 6)(10\beta_2 - 12\beta_1 - 18)^{-1}$$

$$\kappa = \frac{1}{4}c_1^2(c_0c_2)^{-1} = \frac{1}{4}\beta_1(\beta_2 + 3)^2(4\beta_2 - 3\beta_1)^{-1}(2\beta_1 - 3\beta_1 - 6)^{-1}$$

Classification

I: $\kappa < 0$

II: $\beta_1 = 0,\ \beta_2 < 3$

III: $2\beta_2 - 3\beta_1 - 6 = 0$

IV: $0 < \kappa < 1$

V: $\kappa = 1$

VI: $\kappa > 1$

VII: $\beta_1 = 0,\ \beta_2 > 3$

Main types of Pearson system*)

Tip I: $$f(x) = \frac{(x-a)^{p-1}}{b^p\, B(p,q)} \left[1 - \frac{(x-a)}{b}\right]^{q-1}, \quad a \le x \le a+b, \quad b > 0$$

Tip IV: $$f(x) \;=\; \frac{\Gamma(b + b\delta\, i)\, \Gamma(b - b\delta\, i)\, \tau^{2b-1}}{\Gamma(b)\, \Gamma\left(b - \frac{1}{2}\right)\, \pi^{\frac{1}{2}}} \frac{\exp\left[2b\delta \operatorname{arctg}\left(\frac{x-\mu}{\tau}\right)\right]}{\left[(x-\mu)^2 + \tau^2\right]^b},$$

$$-\infty < x < +\infty, \quad \tau > 0,\, b > \frac{1}{2}$$

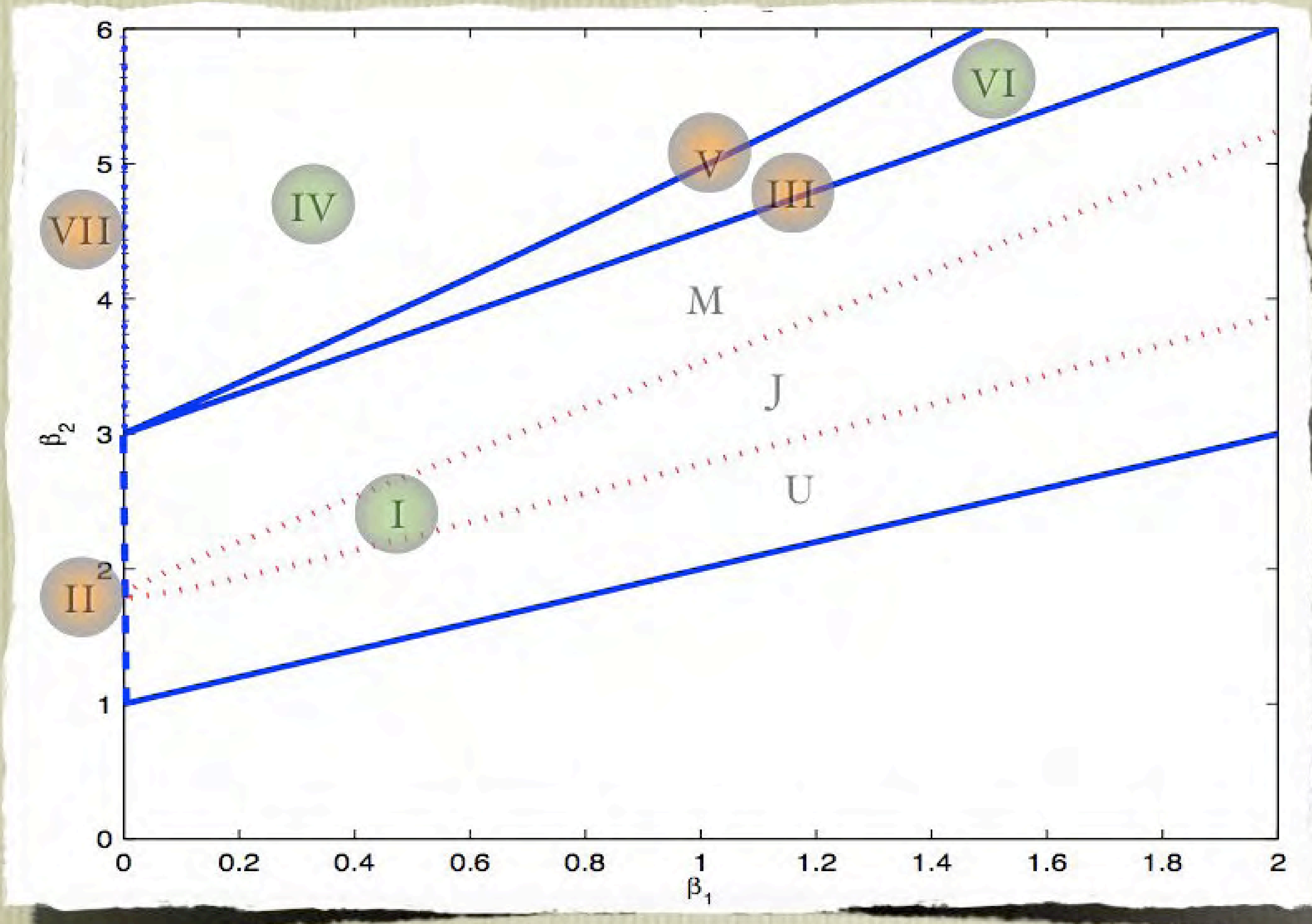
Tip VI: $$f(x) \;=\; \frac{\alpha^m (x-a)^{\beta-1}}{B(\beta, m)\, (\alpha + a - x)^{m+\beta}}, \quad x \ge a, \mathrm{za}\ (\mathrm{As} > 0)$$

$$f(x) \;=\; \frac{\alpha^m (-x+a)^{\beta-1}}{B(\beta, m)\, (\alpha + a - x)^{m+\beta}}, \quad x \le a, \mathrm{za}\ (\mathrm{As} < 0)$$

$$\alpha > 0,\ \beta > 0,\ m > 0$$

*) Nagahara, Y. : *A method of simulating multivariate nonnormal distributions by the Pearson distribution system and estimation*, Computational Statistics & Data Analysis, **47**, 2004

Beta plane (β_1, β_2)



Maximum Likelihood Method

- **Idea*)**: to find those parameters of probability density functions that provide the greatest probability (maximum likelihood) for the occurrence of precisely those values that are measured.

*) First proposed by Sir Ronald Aylmer Fisher (1890-1962) in 1912, at the age of 22, in the article: *On an absolute criterion for fitting frequency curves*, Messenger of Mathematics (1912), **41**, 155-160.

Maximum Likelihood Method

- **Probability:**

  knowing parameters $\rightarrow$
  prediction of outcome

- **Likelihood:**

  observation of data $\rightarrow$
  estimation of parameters

$$f(\mathbf{x}|\boldsymbol{\theta}) \equiv L(\boldsymbol{\theta}|\mathbf{x})$$

prob. density f-on

likelihood f-on

$$\mathbf{x} = (x_1, x_2, \ldots, x_n)^{\mathrm{T}}$$

$$\boldsymbol{\theta} = (\theta_1, \theta_2, \ldots, \theta_k)^{\mathrm{T}}$$
$$\boldsymbol{\theta} = (a, c_0, c_1, c_2)^{\mathrm{T}}$$

Looking for $\boldsymbol{\theta}^*$

- We are looking for $\boldsymbol{\theta}^*$ which maximizes likelihood

$$\mathcal{L}(\boldsymbol{\theta}^*|\mathbf{x}) = \max_{\boldsymbol{\theta}} \mathcal{L}(\boldsymbol{\theta}|\mathbf{x}) = \max_{\boldsymbol{\theta}} \sum_{i=1}^{n} \ln f_i(x_i|\boldsymbol{\theta})$$

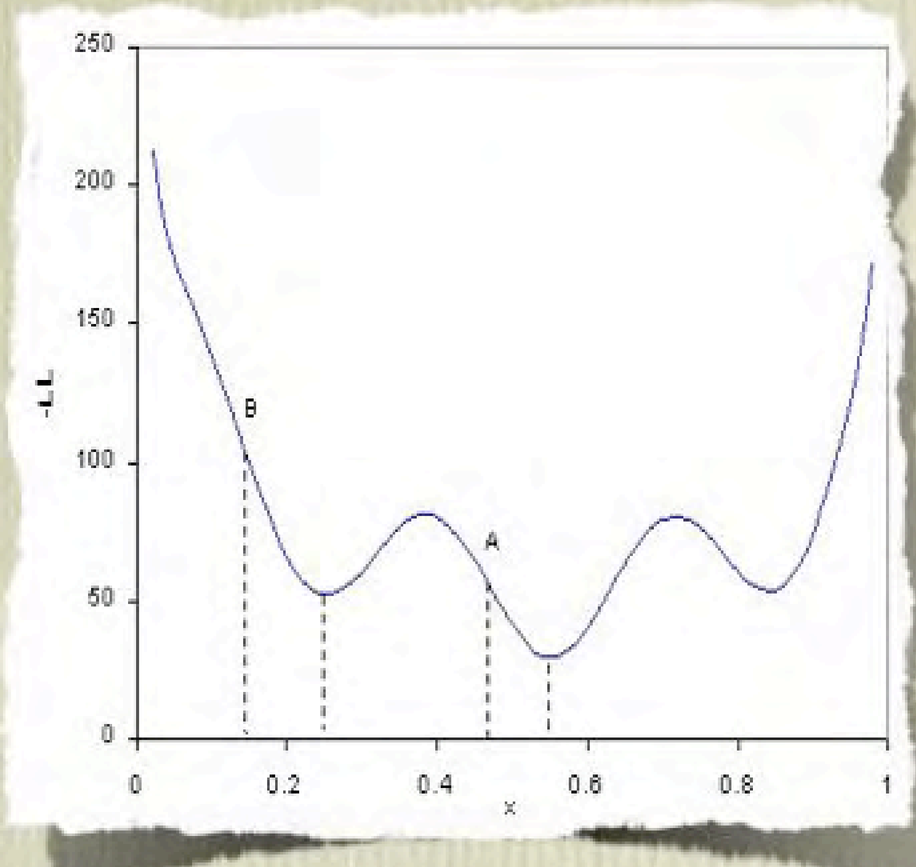
- Because it is not possible to solve this task analytically, we apply numerical optimization methods.

Numerical optimization

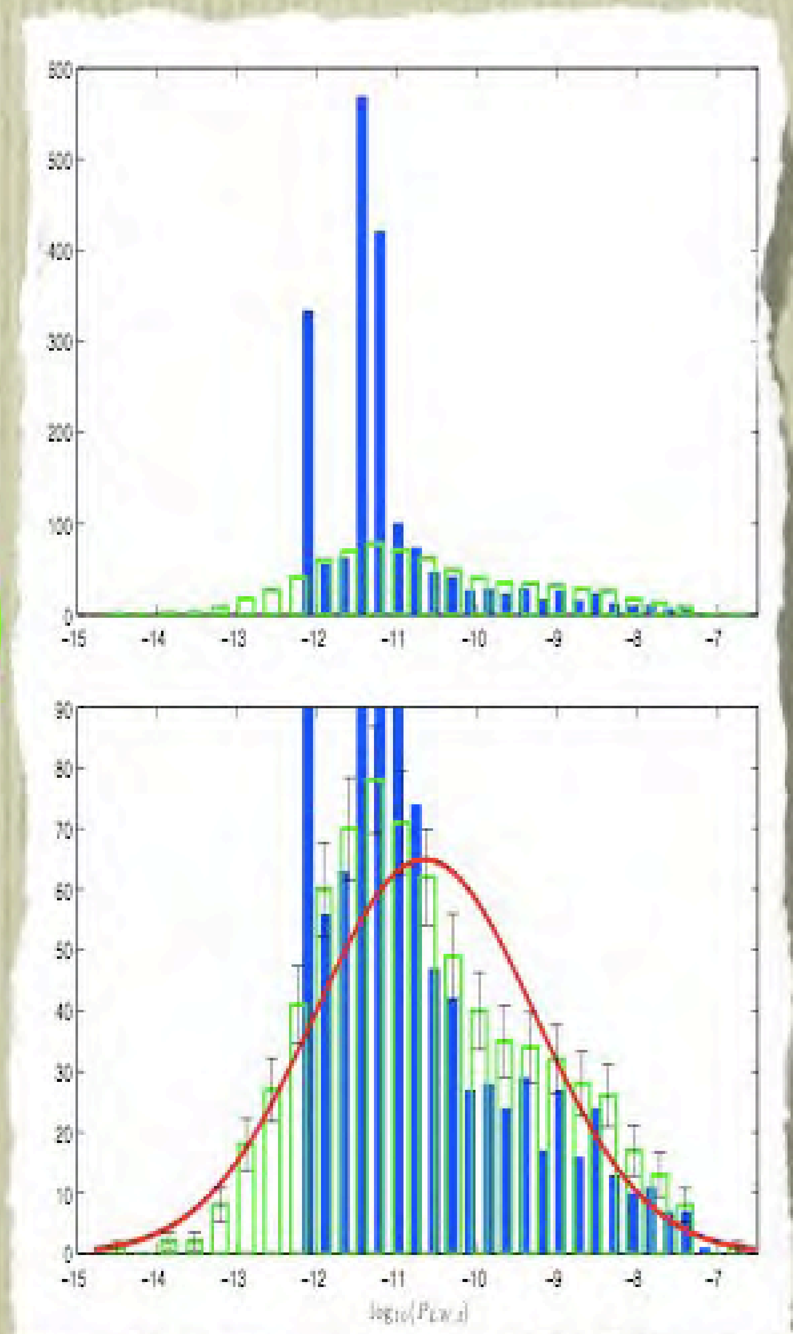
- Methods:
    - Nelder - Mead
    - Levenberg - Marquardt
- It is important to chose good starting values of parameters!

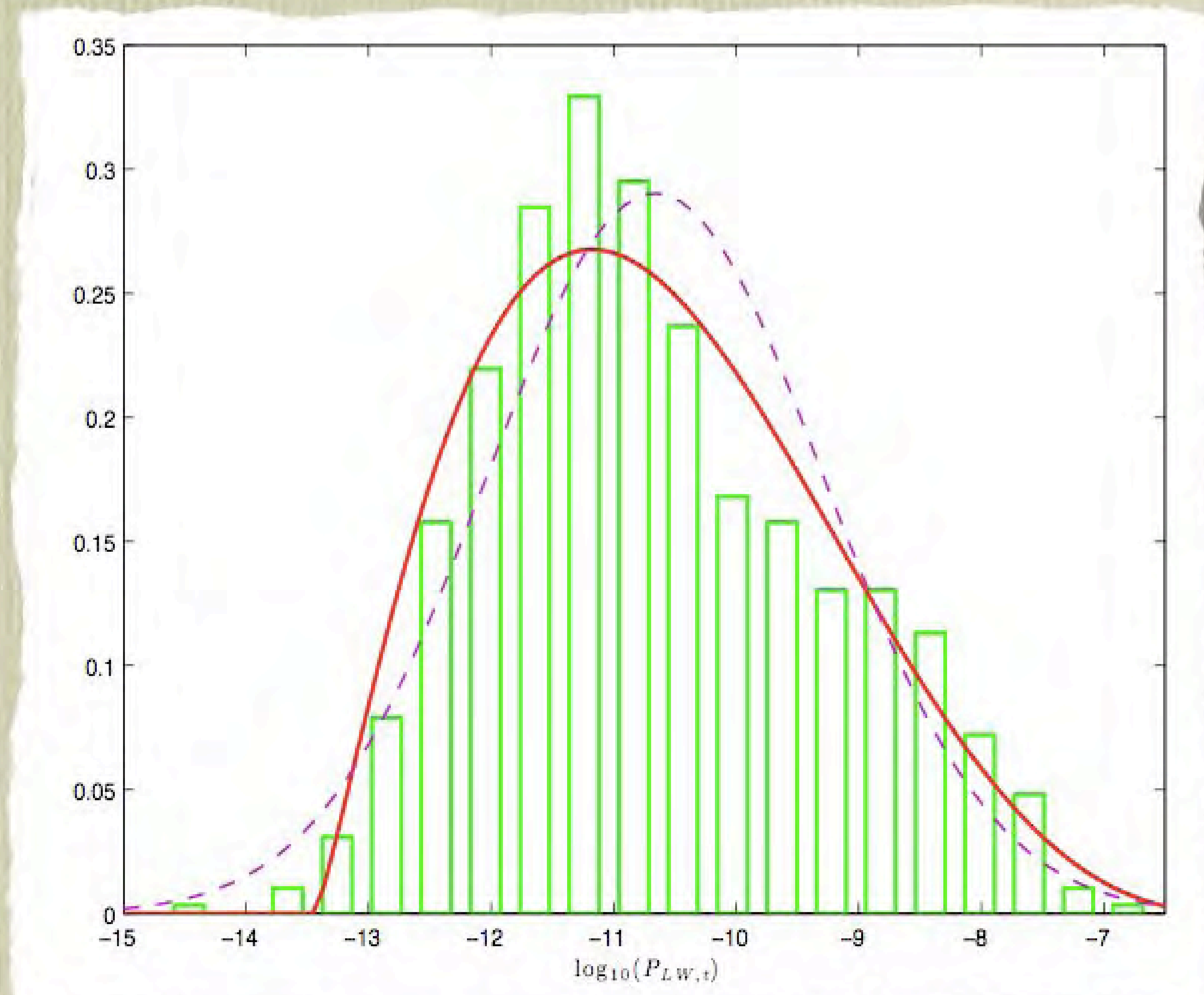
Starting values of parameters are calculated by method of moments.

1D example

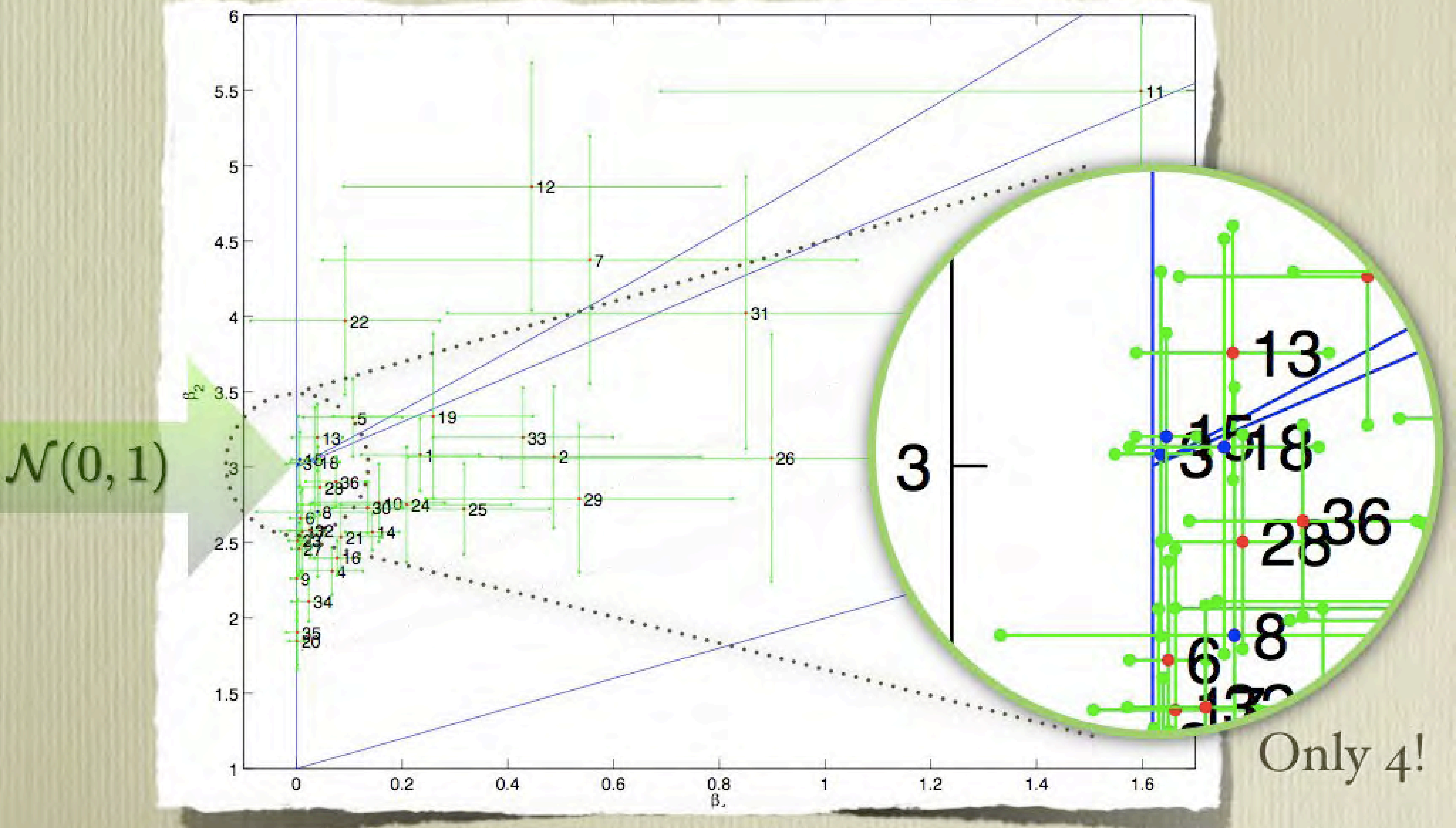


Results





Beta plane (β1, β2)



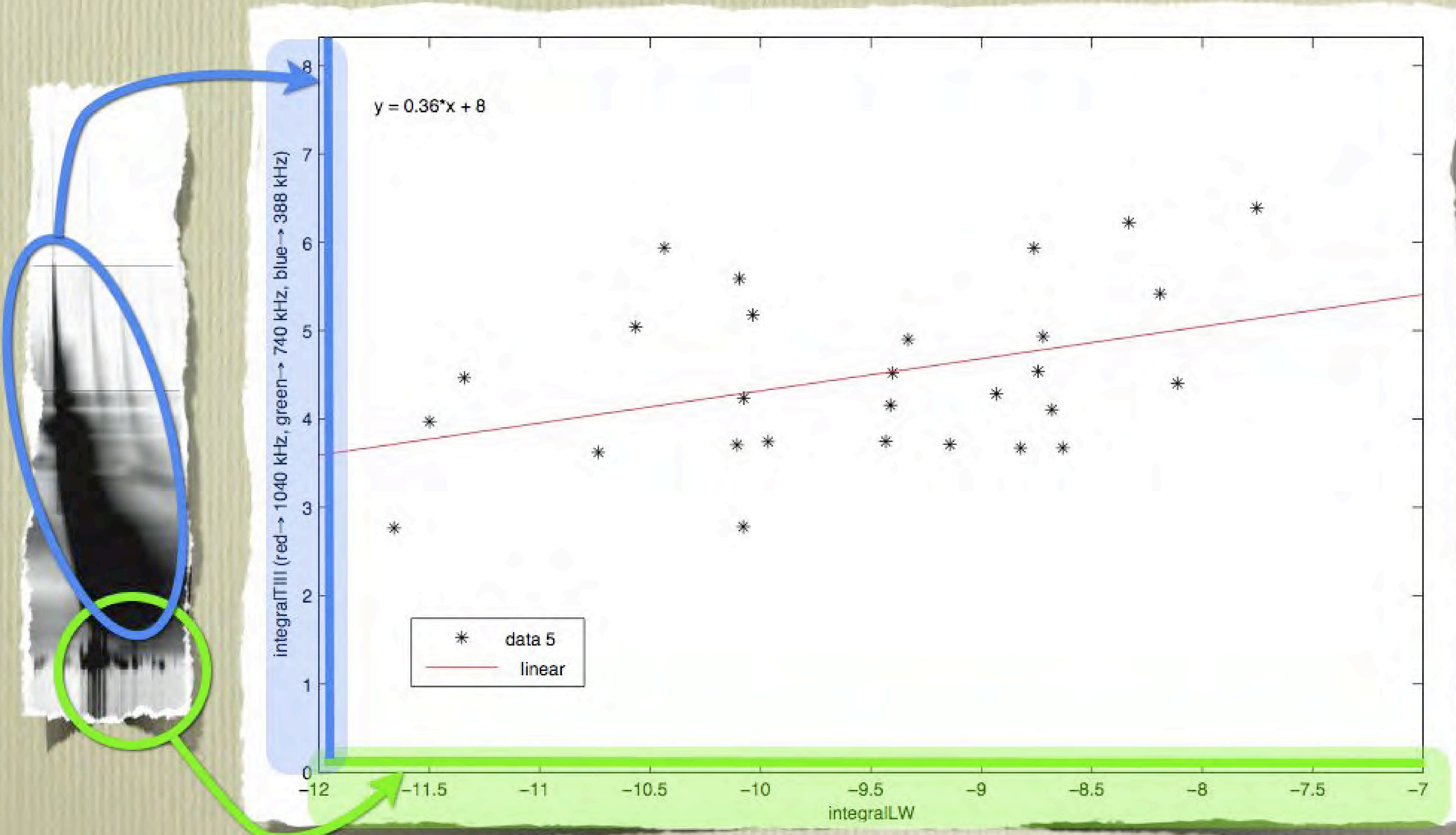
Conclusions

In the examination of 36 events, concerning intensive locally formed Langmuir waves, it is found that:

- all events belong to 3 types of Pearson's distribution: I, IV and VI;
- Pearson's distribution offers better fits ($\chi 2$ test) than a Gaussian ones though the SGT predicts a Gaussian distribution;
- uncertainty analysis goes in favor of Pearson's system of distributions.

Q: What is the reason for disagreement with SGT?
Is it important?
Implications?

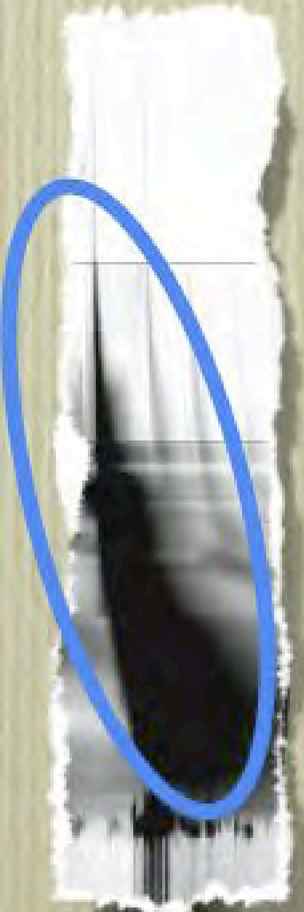
LW & TIII

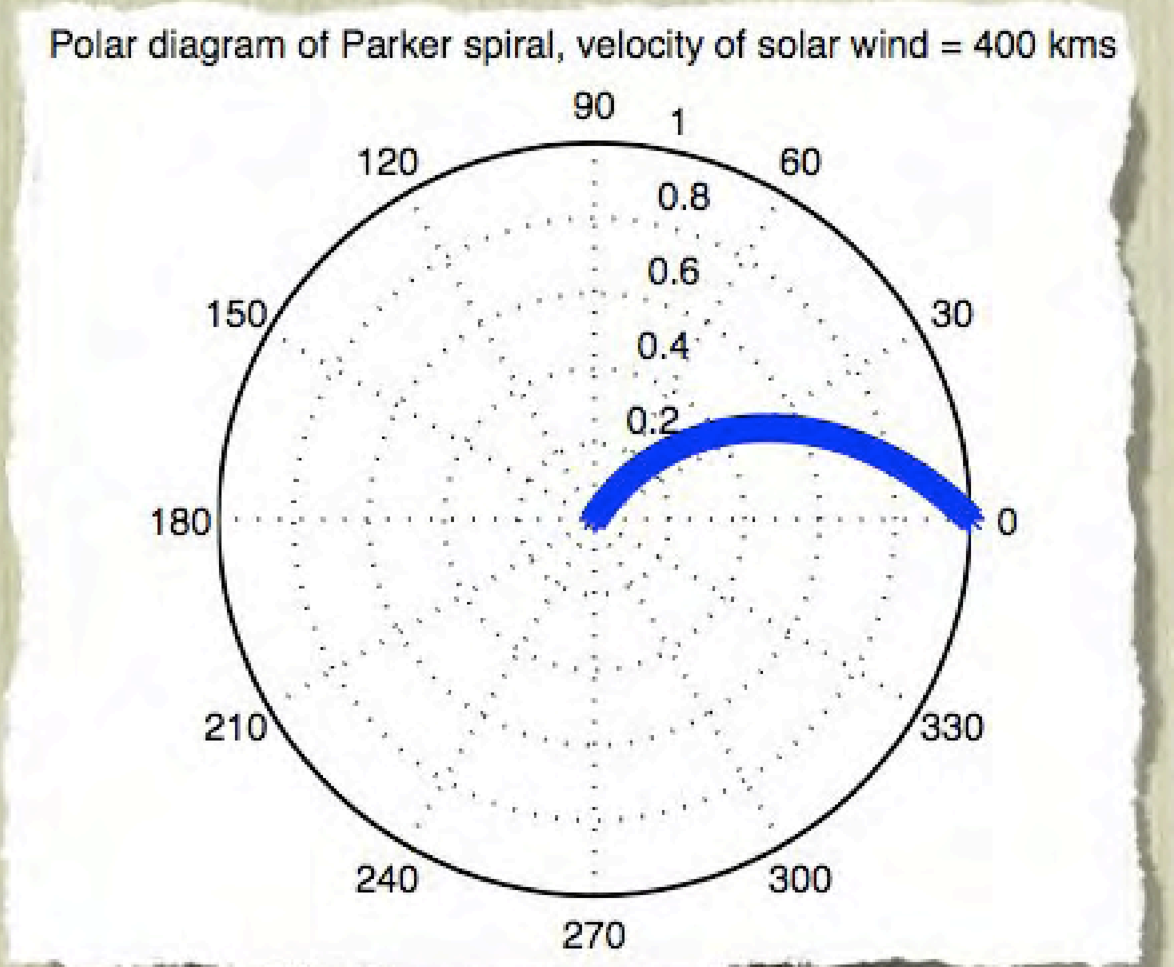


y = 0.36*x + 8
integralTIII (red→ 1040 kHz, green→ 740 kHz, blue→ 388 kHz)
integralLW
data 5
linear

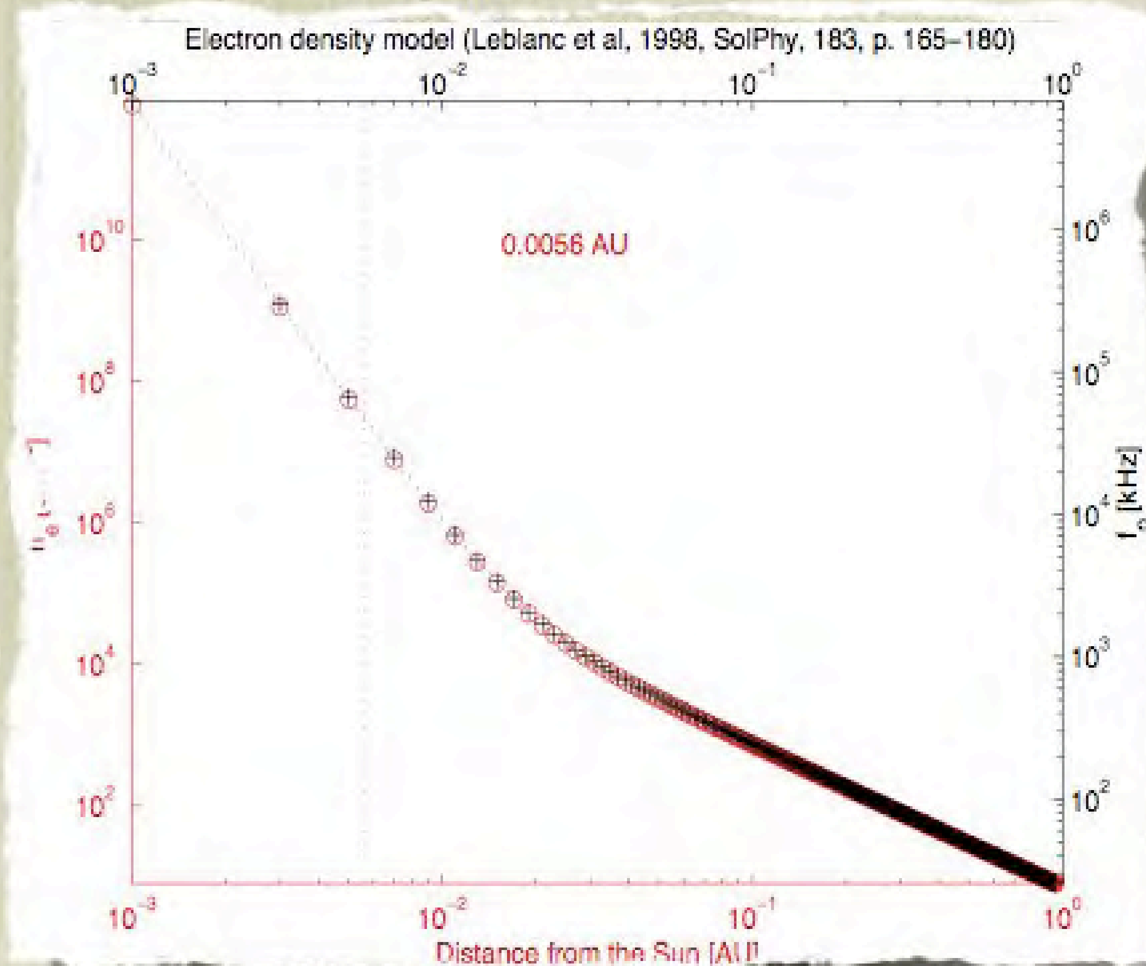
T III

- Directivity, (Bonnin et al, 2008) $D(\varphi) = 10^{a[\cos(\varphi - \varphi_0) - 1]}$





Parker (1954), solar wind model



Leblanc (1998), elect. density model

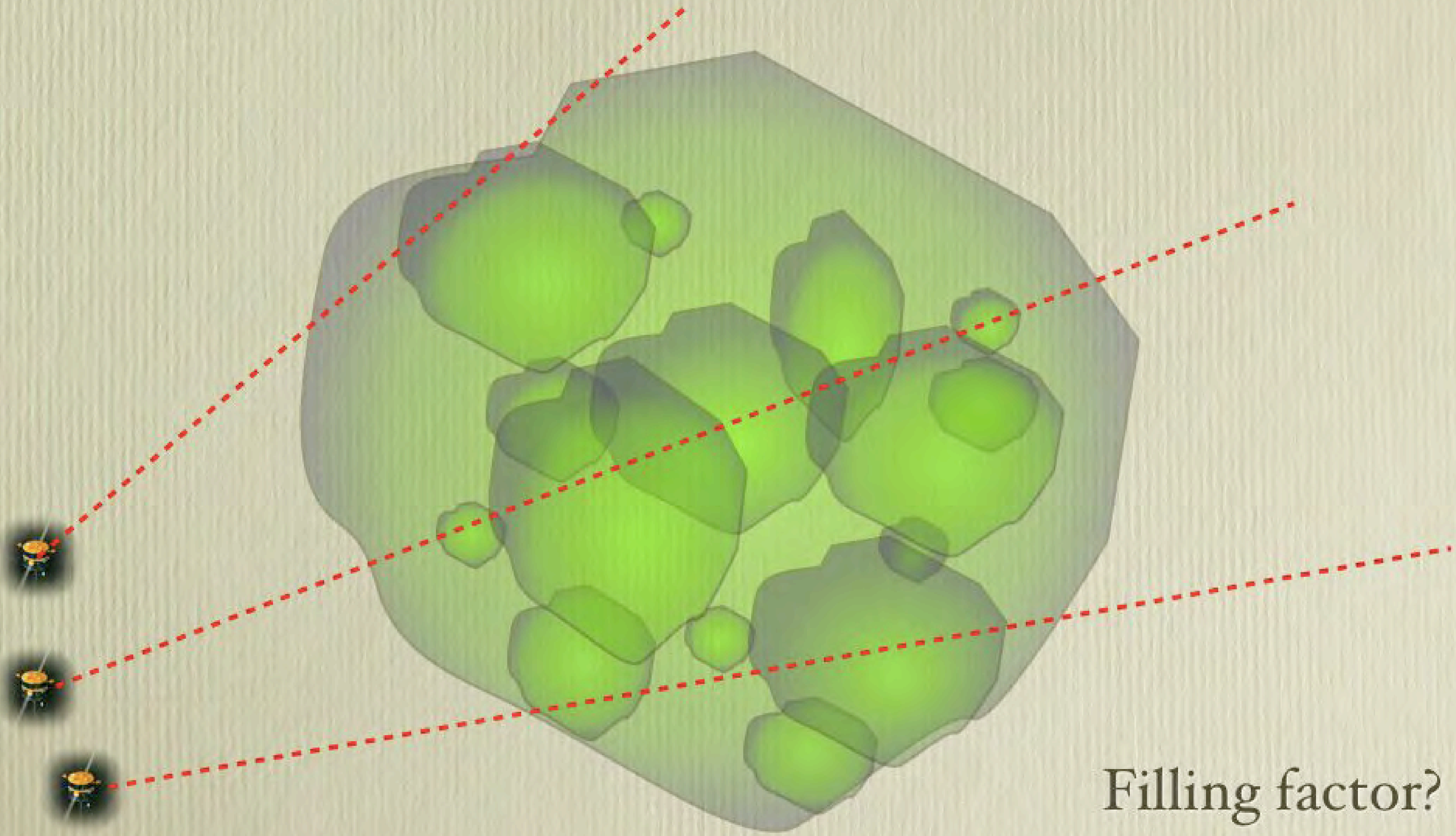
TIII corrections

- Directivity, (Bonnin et al, 2008)

$$D(\varphi) = 10^{a[\cos(\varphi - \varphi_0) - 1]}$$

- Smoothing & Fitting

LW: Beam cross section... ???



Filling factor?

Solar Wind 12, 2009, St. Malo (peer reviewed proceeding)

Langmuir Waves and Type III Bursts Observed by the Wind Spacecraft

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Abstract. Interplanetary electron beams, produced by CMEs and flares, are unstable in the solar wind and generate Langmuir waves at the local plasma frequency or its harmonic. Radio observations of those waves in the range 4 kHz – 256 kHz from the WAVES experiment onboard the WIND spacecraft have been statistically analyzed. A subset of 17 events has been selected for this study. The background consisting of thermal noise, type III bursts and Galactic background has been removed and the histogram of the remaining power spectral density has been fitted by Pearson's system of distributions.

Keywords: Langmuir waves, Type III bursts, Fitting by Pearson's system.
PACS: 96.60.Vg, 95.85.Bh, 95.75.Pq.

Article in preparation with the full set of 36 events

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