Langmuir wave propagation in inhomogeneous plasma

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Langmuir wave propagation

Evolution of Langmuir wave spectral energy density;

$$\frac{\partial}{\partial t}\left(\frac{W(x,k,t)}{\omega(x,k,t)}\right)+v_g\frac{\partial}{\partial x}\left(\frac{W(x,k,t)}{\omega(x,k,t)}\right)+F\frac{\partial}{\partial k}\left(\frac{W(x,k,t)}{\omega(x,k,t)}\right)=Src$$

$v_g = \frac{\partial\omega}{\partial k}$ group velocity *F* force acting on the waves
Src contains all sources/sinks of waves

For inhomogeneous plasma, with density $n_e + \tilde{n}(x,t)$

force on Langmuir plasmon is $F(x,t) = -\frac{1}{2}\omega_{pe}\frac{\partial}{\partial x}\tilde{n}(x,t)$

with ω_{pe} the plasma frequency

Diffusion Term

Can use the quasilinear approximation to rewrite this

term as $$\frac{\partial}{\partial k} D_k \frac{\partial}{\partial k}\left(\frac{W_k}{\omega_k}\right)$$

with D_k the diffusion coefficient

For Langmuir waves, group velocity is $v_g = \frac{3V_{Te}^2 k}{2\omega_{pe}}$

giving D_{vg}

Diffusion Term ctd

For a spectrum of density fluctuations $\tilde{n}(q,\Omega)$

this is $D_{v_g} = \frac{1}{4\pi}\int_{-\infty}^{\infty} dq \int_{-\infty}^{\infty} d\Omega q^2 \tilde{n}^2(q,\Omega)\delta(\Omega - qv_g)$

Exact form of diffusion coefficient depends on spectrum of density fluctuations

For example, ion sound waves could be source;

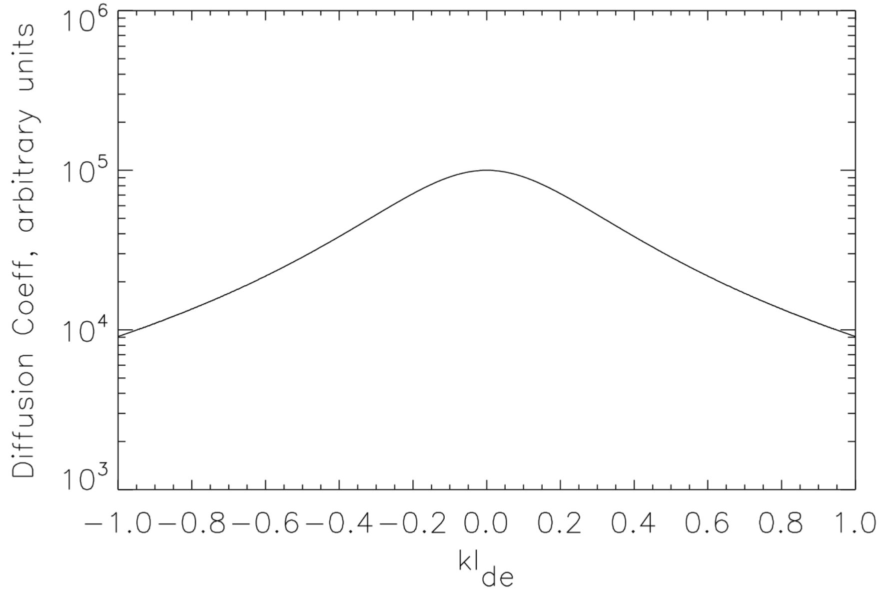
dispersion relation $\Omega = \frac{qv_s}{\sqrt{1+q^2\lambda_{De}^2}}$

Diffusion Coefficient ctd

Gaussian spectrum $$\tilde{n}(q,\Omega) = \tilde{n}_0^2 \exp\left(\frac{-q^2}{k_0^2} - \frac{\Omega^2}{\omega_0^2}\right)$$

gives coefficient $$D_{v_g} = \sqrt{\pi}\frac{\tilde{n}_0^2}{2}\left(1 + \frac{v_g^2}{v_0^2}\right)^{-3/2} k_0^3$$

where $v_0 = \omega_0 / k_0$



Quasilinear equations; 1D

- Source of density fluctuations gives their spectrum
- OR propose generic spectrum
- From this, calculate diffusion coefficient

$$\frac{\partial W_k}{\partial t} = \frac{\pi \omega_{pe}}{n_e} v^2 W_k \frac{\partial f(v)}{dv} + \frac{\omega_{pe}^3 m_e}{4\pi n_e} v \ln\left(\frac{v}{v_{Te}}\right) f(v) - \frac{\gamma_c}{4} W_k + \frac{\partial}{\partial k} D_k \frac{\partial}{\partial k}\left(\frac{W_k}{\omega_k}\right)$$

$$\frac{\partial f}{\partial t} = \frac{\pi \omega_{pe}^2}{n_e m} \frac{\partial}{\partial v}\left(\frac{W_k}{v} \frac{\partial}{\partial v} f(v)\right) + \gamma_c \frac{\partial}{\partial v}\left(\frac{f}{v^2} + \frac{v_{Te}^2}{v^3}\frac{\partial f}{\partial v}\right)$$

Terms;
stimulated and spontaneous emission; collisional damping; diffusion
emission; collisional damping

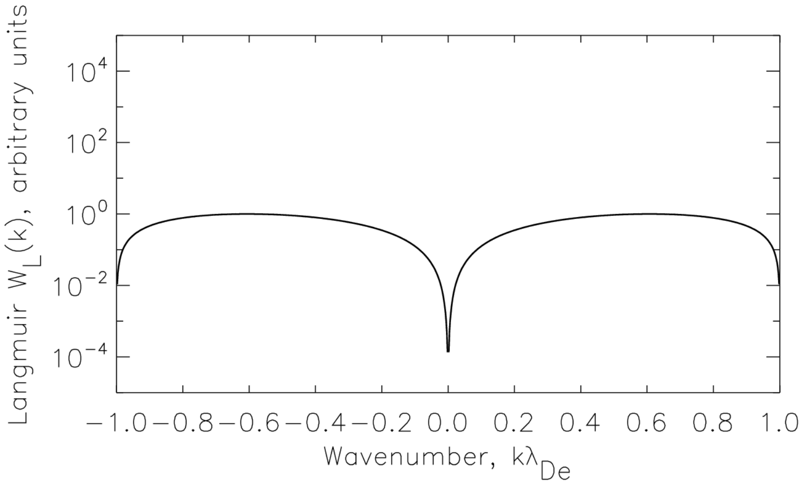
Thermal level of L waves

In thermal equilibrium the electron spectrum Maxwellian

Corresponding thermal level of Langmuir waves;

$$\frac{\pi \omega_{pe}}{n_e} v^2 W_k \frac{\partial f(v)}{dv} + \frac{\omega_{pe}^3 m_e}{4\pi n_e} v \ln\left(\frac{v}{v_{Te}}\right) f(v) = 0$$

$$W_k = \frac{T_e}{4\pi^2} k^2 \ln\left(\frac{1}{k\lambda_{De}}\right)$$

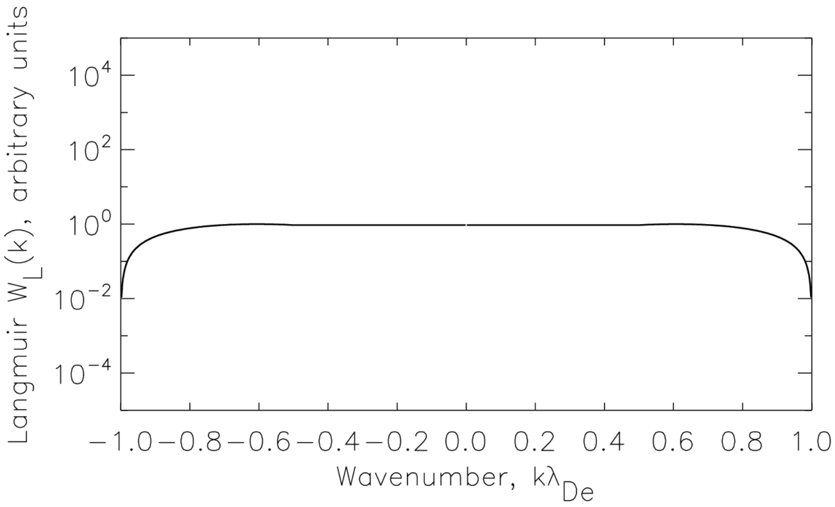


Flat L wave spectrum

Diffusion term is zero when $\frac{\partial}{\partial k}\left(\frac{W_k}{\omega_k}\right) = 0$

So for sufficiently large diffusion coefficient, effect is to flatten Langmuir spectrum.

"Best" possible effect is;



Kappa distribution

For Langmuir wave spectrum constant for all *k,* the electron distribution function will be a kappa distribution;

$$f(v) \propto \left(\frac{v^2}{\kappa v_{Te}^2} + 1\right)^{-\kappa-1}$$

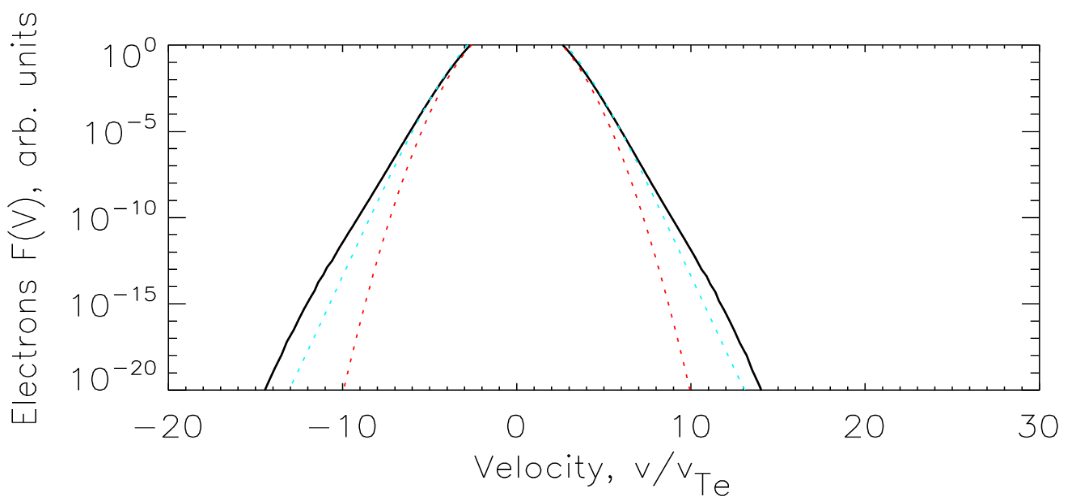
but for the L wave spectrum just shown, κ~50 and the spectrum is close to Maxwellian

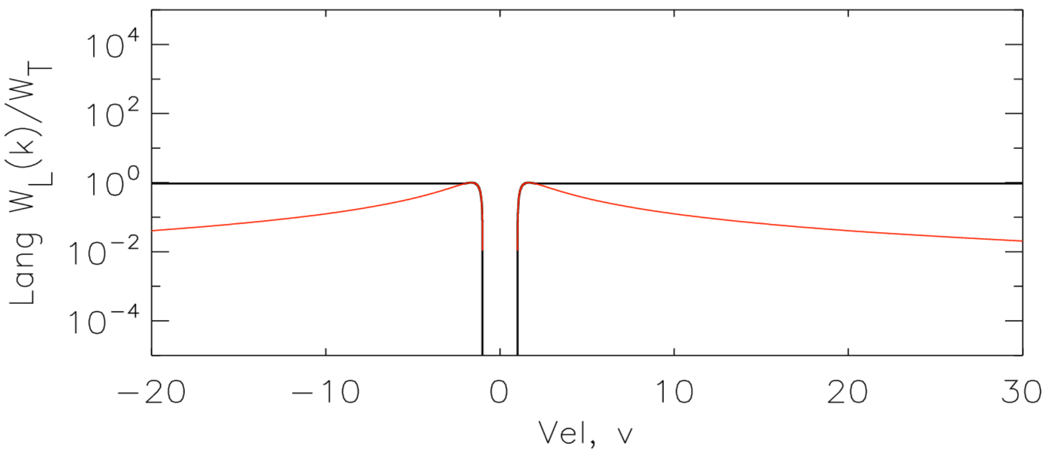
Simulation results

Simulation result for this Langmuir spectrum;

Part of this is numerical effect from code

Predicted kappa distribution in pale blue





More significant effects?

Effect of diffusion on purely thermal Langmuir waves is minor

Source of Langmuir waves will increase effect

From electrons;

Beam is obvious source

Strahl electrons possible?

Other sources?

End

Thanks for your attention!